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Concept-cognitive learning survey: Mining and fusing knowledge from data

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ABSTRACT

Concept-cognitive learning (CCL), an emerging intelligence learning paradigm, has recently become a popular research subject in artificial intelligence and cognitive computing. A central notion of CCL is cognitive and learning things via concepts. In this process, concepts play a fundamental role when mining and fusing knowledge from data to wisdom. With the in-depth research and expansion of CCL in scopes, goals, and methodologies, some difficulties have gradually emerged, including some vague terminology, ambiguous views, and scattered research. Hence, a systematic and comprehensive review of the development process and advanced research about CCL is particularly necessary at the moment. This paper summarizes the theoretical significance, application value, and future development potential of CCL. More importantly, by synthesizing the reviewed related research, we can acquire some interesting results and answer three essential questions: (1) why examine a cognitive and learning framework based on concept? (2) what is the concept-cognitive learning? (3) how to make concept-cognitive learning? The findings of this work could act as a valuable guide for related studies in quest of a clear understanding of the closely related research issues around concept-cognitive learning.

1. Introduction

Recently, the emergence of new technologies such as Gemini[1] and ChatGPT[2] has brought attention to artificial intelligence and cognitive computing. Simultaneously, the question of how to define a notion of “intelligence” has attracted concentration in academia and industry again. In fact, cognitive computing and artificial intelligence initially had the same goals, both involving the study of computational modeling of the human brain, including memory, attention, perception, reasoning, planning, decision-making, etc. Hence, a close relationship exists between both fields. Investigating various valuable cognitive intelligence paradigms is essential for studying and enforcing the fundamental assumptions of artificial intelligence. Despite remarkable progress in simulating human logical thinking, the credibility and reliability of artificial general intelligence (AGI) have eluded expectations of the intelligence machine system. According to Lake et al. [1], “People learning new concepts can often generalize successfully from just a single example, yet machine learning algorithms typically require tens or hundreds of examples to perform with similar accuracy”. The research of machine learning should focus on the concept learning of human-level conceptual knowledge. In this sense, cognitive learning of concepts is undoubtedly a valuable cognitive intelligence paradigm.

In general, the goal of data processing and analysis is to form concepts and rules that guide human decision-making behavior. Among these, concepts are the most fundamental unit of human cognition in philosophy, which refers to the common essential characteristics of things abstracted and summarized by human beings in the process of understanding and knowing the world [2,3]. With the help of concepts,

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[1] https://deepmind.google/technologies/gemini/
[2] https://openai.com/blog/chatgpt

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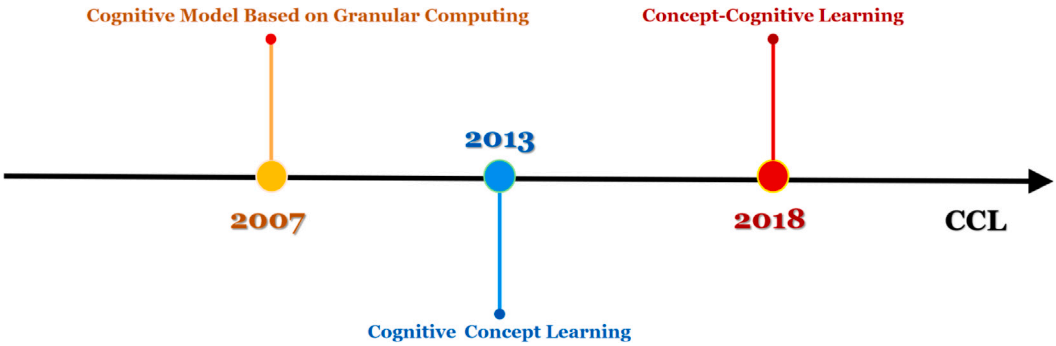

Fig. 1. Development stage of CCL.

humans can build a mapping between abstraction and reality and explore the development law of things while recognizing things, that is cognition. Note that the concept mentioned above is a broad concept but also a standard concept. That is, it has three elements, including annotation (i.e., name of concept), connotation (i.e., definition of concept), and denotation (i.e., referent of concept). For example, consider the concept of a prime number, "*A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself*". Here, the annotation is "*prime number*", the connotation is "*the natural number greater than 1 that has no positive divisors other than 1 and itself*", and the denotation is "*the set of natural numbers* $\{2, 3, 5, 7, 11, 13, \ldots\}$" that satisfy this definition.

The study of concept representation and learning is a noteworthy topic that spans various disciplines, such as philosophy, mathematics, artificial intelligence, and cognitive science, among others. Formal concept analysis (FCA) was proposed by Wille [4] in 1982 as a means of formally describing concepts. The core idea of FCA involves expressing objects, attributes, and the relations between objects and attributes based on a formal context that comprises the ontology in a structured manner. In a formal context, connotation is also known as intent, and denotation is also known as extent. One enables the construction of all concepts and their generalization and specialization relationships, forming a concept lattice for the clear expression of the knowledge structure. In this way, concepts can be concretely expressed in various forms (including formal concepts, prototype concepts, granular concepts, and fuzzy concepts) to describe the semantic interpretation of the ontology in different scenarios, such as medical diagnosis, handwritten numeral, micro-expressions, etc. In particular, in medical diagnosis, paper [5] designed a concept-cognitive learning system for genetic data analysis and successfully applied it to the task of tumor diagnosis. This system utilizes a fuzzy-based concept-cognitive learning model to analyze tumor diagnosis from the perspective of gene analysis, which effectively improves the medical diagnosis accuracy of tumor patients.

During the past 42 years, we have witnessed a growing interest and development of formal concept analysis [6–12]. At the same time, this theory focuses more on the construction of concept lattice, which is an NP-hard problem and does not suit the area of big data, especially in the area of artificial intelligence. As mentioned above, people learning a new concept can often generalize successfully from very few examples, and machine learning algorithms typically require tens or hundreds of examples to perform with similar accuracy. Therefore, cognitive learning for concepts is emerging in artificial intelligence and cognitive science. An emerging intelligence paradigm via concept learning, coined by Zhang and Xu [13] in 2007, is an influential tool for researching concept learning and cognitive intelligence. This work mainly discusses the sufficient and necessary relationship between objects and attributes. Then, the unity of objects and attributes forms the concept, which is a two-way learning process as the essence of the cognitive process. Based on this essence, a concrete mathematical model of cognition is presented, along with a detailed description of cognitive granulation. Since then, the rudiments of cognitive concept learning come into being. Moreover, Wang [14] proposed an important concept algebra viewpoint for cognitive learning. As a complement to concept learning, Yao [15] interpreted concept learning from the perspective of cognitive informatics and granular computing and pointed out that cognitive concept learning should be carried out from three sub-levels: the philosophy level, the algorithm/technique level, and the application level. Undoubtedly, these important studies have laid the foundation for research on cognitive learning via concepts. For this reason, these studies are also called the rudiments of cognitive concept learning.

Early research viewed cognitive concept learning as the process of learning concepts through specific cognitive methods and uncovering cognitive learning rules within the human brain. From 2013 to 2018, with the rise of cognitive concept learning, numerous scholars continued to explore this area and developed various cognitive concept learning models and methods. One notable research line is focused on a cognitive concept learning model based on granular computing. For instance, Xu et al. [16,17] described the concept learning process through sufficient and necessary learning, also known as two-way concept learning (TCL). Kumar et al. [18] utilized formal concept analysis to investigate the cognitive functionalities of bidirectional associative memory. Li et al. [19] proposed a granular concept learning model using granular computing from a cognitive viewpoint. Shivhare et al. [20] introduced a three-way conceptual model to cognitive memory functionalities. Although these studies significantly contributed to the advancement of cognitive concept learning, it is mainly limited within the framework of granular computing.

With the in-depth research and expansion of concept-cognitive learning in scopes, goals, and methodologies, researchers in various fields have researched this subject from their professional perspectives. There were various theoretical frameworks in the same field, such as medical diagnostics [5], knowledge discovery [21], pattern recognition [22], cloud computing [23], online learning [24], sentiment analysis [25], and others. In 2018, the term "concept-cognitive learning" was widely accepted and recognized. Since 2007, the development stage of the CCL term is shown in Fig. 1. In particular, the fruitful marriage with machine learning further broadens the research horizon for this field. Different from formal concept analysis and cognitive concept learning, concept-cognitive learning, the science of cognition and learning things via concepts, aims to explore human-level information processing and conceptual-knowledge learning mechanisms from a cognitive viewpoint, with planned applications in studying and implementing human-like intelligent systems. The process of knowledge mining and fusion in concept-cognitive learning relies on beginning with a specific formal context, namely a cross table, where each row corresponds to a set of objects, each column refers to a set of attributes, and the values in the cross table indicate the relationship between objects and attributes. Hence, in recent years, increasing attention has been directed toward CCL in data mining, intelligence computing, cognitive computing, and intelligence decision-making. The core concerns of concept-cognitive learning include the concept

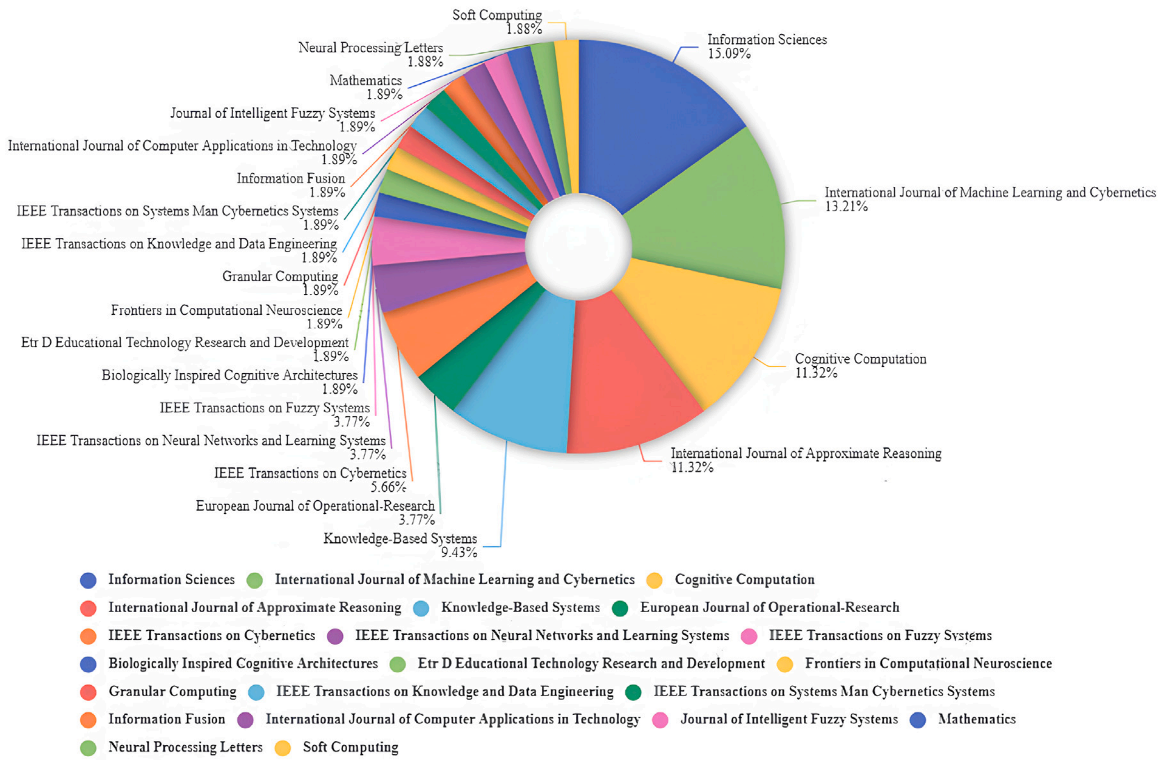

Fig. 2. Publications of CCL.

cognition mechanism, concept learning method, cognitive system construction mechanism, complex decision optimization mechanism, and others. Specific concept-cognitive learning models include memory-based CCL [2], incremental CCL [26], fuzzy-based CCL [27], two-way CCL [28], semi-supervised CCL [29], etc. A detailed analysis of various CCL models can be found in Ref. [30], and the clear developmental background of CCL can be seen in Ref. [31]. In addition, the first book on concept-cognitive learning was also published in 2023 by Xu et al. [32]

Concept-cognitive learning, an emerging intelligence learning paradigm, has recently become a popular research subject in artificial intelligence and cognitive computing. Nevertheless, some issues have gradually emerged, including some vague terminology, ambiguous views, and scattered research. Consequently, a systematic and comprehensive summary of the development process, advanced research, and future development of concept-cognitive learning is particularly necessary, especially in answering some essential questions: (1) why examine a cognitive and learning framework based on the concept? (2) what is the concept-cognitive learning? (3) how to make concept-cognitive learning? In this article, we review the published papers related to CCL and overview the research thought and the representative methods of CCL. The main contributions of this paper are as follows.

- It is the systematic overview that attempts to provide an in-depth analysis of the advancement of concept-cognitive learning. One combs a comprehensive analysis and valuable reference for related research according to the analysis of publication articles, the basic theory of CCL, the categorization of CCL, challenges, and future directions of CCL.
- It is a multi-view categorization of concept-cognitive learning from the three levels of abstract-machines-brain, which surveys several triadic structures for characterizing CCL, namely, the information-processing triangle, the three research scopes (i.e., mathematics and logics, artificial intelligence, and cognitive simulations), the three research goals (i.e., concept analysis methods, concept learning strategy, and concept cognitive mechanism), and three research methodologies (i.e., cognitive computing, granular computing, and machine learning).
- It is an elucidation of the main research gaps and suggestions for future research directions for the model, method, and application of concept-cognitive learning from six aspects: concept learning method, concept cognition mechanism, cognitive system construction and optimization, complex decision-making, interdisciplinary research, and engineering applications.
- It acquires some interesting results by synthesizing the reviewed related research. These findings could act as a valuable guide for related studies in quest of a clear understanding of the closely related research issues around concept-cognitive learning.

Furthermore, three essential questions in this review can be answered in six sections. In Section 1 and Section 2, we focus on replying to the first question in the motivation, i.e., why do we examine a cognitive and learning framework based on concept? Section 3 answers the question of what concept-cognitive learning is and some notions related to it. As for the last question, Section 4 overviews the existing methodologies on different topics related to CCL, and future directions about CCL are discussed in Section 5. Finally, the concluding remarks are presented in Section 6.

2. Analysis of published articles

In this section, we will unveil a clear connection among keywords, authors, affiliations, papers, and journals. Through a comprehensive analysis of the dataset comprising concept-cognitive learning articles published before February 1, 2024, and downloaded from ISI Web of Science using the keywords "cognitive concept learning" and "concept-cognitive learning". We offer an overview of 53 CCL papers analyzed, including 6 highly cited ESI papers [17,19,27,30,31,33], which represent 11.3% of the total articles.

To clearly show the publication information, we offer statistics of the publication title of the collected papers as a function of publication distribution shown in Fig. 2, and recorded the details of these 6 ESI

Table 1
ESI highly cited paper of CCL in the last decade.

Reference	Author	Title	Citations	Year
[31]	Xu, Guo, Qian & Ding	Two-way concept-cognitive learning method: A fuzzy-based progressive learning	37	2023
[30]	Xu, Guo, Mi, Qian, Ding & Zheng	Two-way concept-cognitive learning via concept movement viewpoint	15	2023
[27]	Mi, Shi, Li, Liu &Yan	Fuzzy-based concept learning method: Exploiting data with fuzzy conceptual clustering	48	2022
[33]	Li, Huang, Qi, Qian & Liu	Three-way cognitive concept learning via multi-granularity	353	2017
[17]	Xu & Li	Granular computing approach to two-way learning based on formal concept analysis in fuzzy dataset	259	2016
[19]	Li, Mei, Xu & Qian	Concept learning via granular computing: A cognitive viewpoint	280	2015

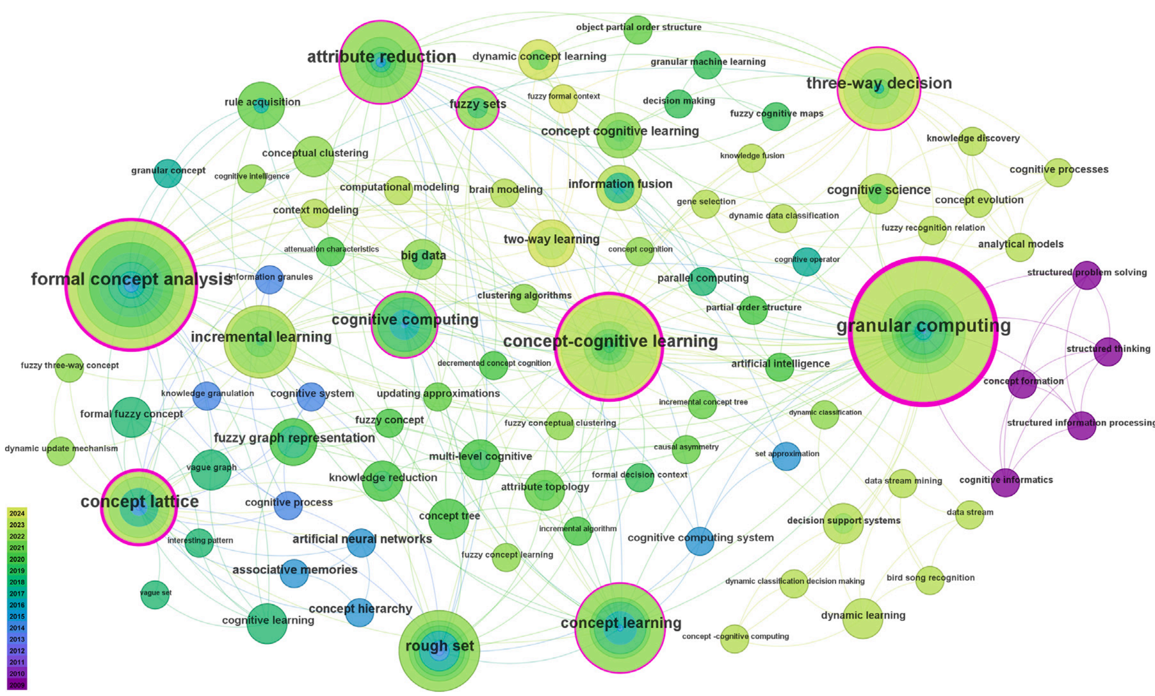

Fig. 3. Keywords network of CCL.

highly cited papers in Table 1 for easy access. It can be seen from Fig. 2 that CCL has been published in 21 famous academic journals, among which the journals of *Information Sciences*, *International Journal of Machine Learning and Cybernetics*, *Cognitive Computations*, *International Journal of Approximate Reasoning* and *Knowledge-Based Systems* are the top five journals in terms of the current number of publications. In addition, it is noted that the latest progress of CCL has also been published successively in the journal of *Information Fusion*, *IEEE Transactions on Neural Networks and Learning Systems*, *IEEE Transactions on Fuzzy Systems*, *IEEE Transactions on Knowledge and Data Engineering*, *IEEE Transactions on Cybernetics*, etc.

In order to gain a better understanding of the research landscape in concept-cognitive learning, this section constructs a network structure diagram that encompasses authors, institutions, and keywords. The diagram is based on 53 papers related to CCL and aims to explore the interconnectedness among different research frontiers. To visually represent the analysis, the diagram employs "growth rings" as nodes, which vary in size and color, indicating different attributes. The colors within each "ring" signify the author or institution associated with the research, while the keywords are categorized by year, transitioning from cool colors to warm colors. The thickness of each growth ring corresponds to the frequency of the respective keyword. A thicker growth ring represents a larger node, indicating a higher frequency of the keyword's occurrence. The lines connecting the nodes represent co-occurrence relationships, with the color of the lines indicating the earliest year in which both nodes appeared together in an article. Besides, the centrality of a node reflects the proportion of all shortest paths passing through that particular point in the network. Nodes with high centrality act as key bridges within the network, establishing strong connections with other nodes. Purple circles denote these key points.

Fig. 3 displays the keyword clustering graph, where each node represents a keyword found in the papers. If two keywords appear in the same paper, there will be a line connecting them. The size of the node indicates the frequency of occurrence of the keyword. In this figure, keywords such as concept-cognitive learning, granular computing, three-way decision, and formal concept analysis are depicted in larger circles, indicating their frequent occurrence. Meanwhile, the outer ring of the concept-cognitive learning, three-way decision, formal concept analysis, and concept learning nodes is purple, highlighting their high centrality and reflecting the current research focus. Furthermore, concept-cognitive learning connects closely with granular computing,

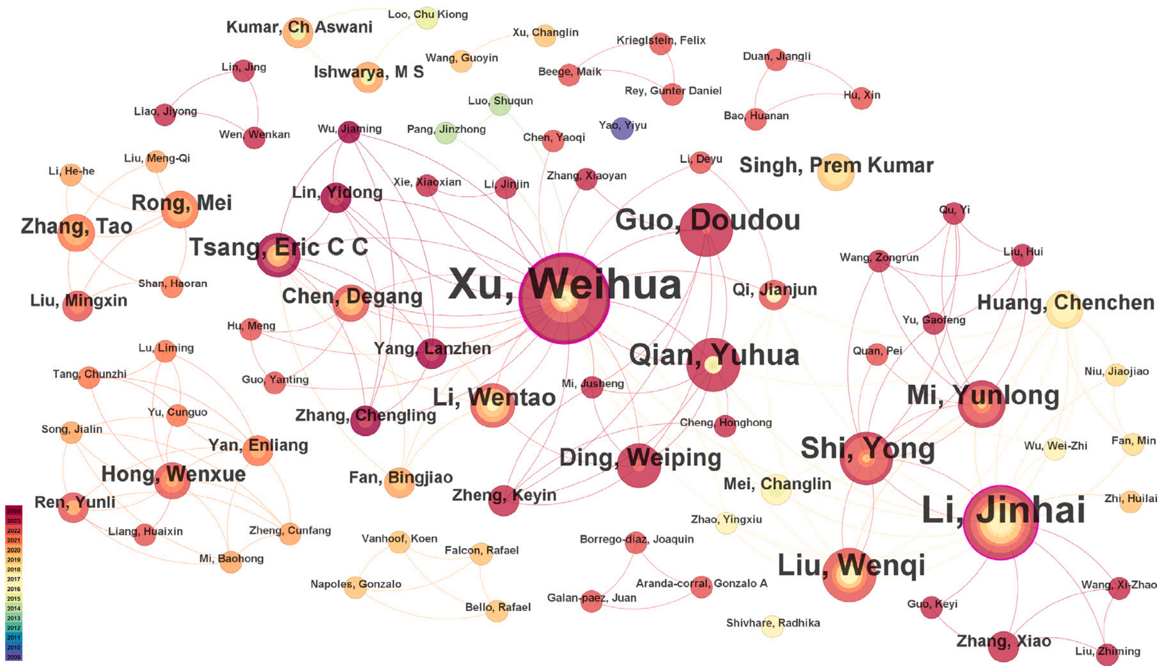

Fig. 4. Scientific collaboration network of CCL.

cognitive computing, and other keywords in machine learning, such as knowledge, attribute reduction, and incremental learning, which also reflects the main research methodologies of CCL at present. Notably, granular computing with high centrality has emerged as a key area of research methodology within concept-cognitive learning.

Fig. 4 shows the author collaboration network diagram, consisting of 88 nodes, where each node represents the author and the lines between the nodes indicate that the author has a collaborative relationship. The larger the node, the greater the number of authors in the field. The more connections a node has with other nodes, the closer the author of the node is to its node and the greater its influence. From this figure, it can be found that Xu Weihua and Li Jinhai have purple outer rings with high centrality, indicating that they often co-appeared with other authors in the same literature and played a bridging role. Xu Weihua has the largest node, and according to statistics, he has 18 co-authored papers, ranking first. In particular, Xu Weihua, Guo Doudou, Qian Yuhua, and Ding Weiping nodes show deep purple, indicating that they have a large number of papers in the past three years and are in a leading position. In addition, Li Jinhai also published 12 articles, accounting for 23% of the statistics. From this figure, one can easily see that there are several non-connected components, for example, groups labeled by Xu Weihua, Li Jinhai, Zhang Tao, and Hong Wenxue. This suggests that further efforts are needed to bring more interaction between these different groups.

Fig. 5 shows the institutional collaboration network diagram with 49 nodes representing different institutions, where the size of each node and the color of the "ring" in the nodes represent the number of publications and the publication time by the respective institution in the current field. The connections between these nodes indicate cooperative relationships between institutions. From this figure, there are three main research groups represented by Southwest University, Kunming University of Science & Technology, and Yanshan University. Particularly, Southwest University boasts the largest node with a frequency of 15 and maintains close connections with other institutions. From this figure, the University of Regina, with the purple node, first attended to the thought about concept-cognitive learning, which has been continuously advanced and researched by different institutions. Currently, papers from Southwest University, led by Xu Weihua and Guo Doudou, have played a prominent role in the field. Additionally, Kunming University of Science and Technology and the Chinese Academy of Sciences have frequencies of 12 and 7, respectively, positioning them as the second and third highest among all institutions.

Furthermore, some interesting observations can be drawn from these figures:

- The keywords network demonstrates the relationships between various concept-cognitive learning topics. As research continues to advance, concept-cognitive learning, granular computing, three-way decision, concept learning, formal concept analysis, and cognitive computing have emerged as hot topics in the field of concept-cognitive learning.
- The scientific collaboration network shows that most papers on concept-cognitive learning are collaborated by multiple researchers. Xu Weihua, Guo Doudou, Qian Yuhua, Ding Weiping, Li Jinhai, and Mi Yunlong have played important roles in the development of concept-cognitive learning, with a particular focus on the contributions of Xu Weihua and Li Jinhai in recent years.
- The university collaboration network reveals that the research organizations of concept-cognitive learning are closely connected, with a concentration of organizations based in China. Southwest University, Kunming University, Yanshan University, and the Chinese Academy of Science are among the organizations actively promoting the development of concept-cognitive learning.

3. Theory of concept-cognitive learning

Within the CCL theory of this section, we provide a brief review and necessary statements of some notions related to concept-cognitive learning, including formal context, formal concept, and basic thoughts. A formal context is especially accustomed to representing the data to be analyzed and then extracting concepts of different levels and the relationships between contexts from a formal context. However, many researchers in research outside CCL and FCA are unaware of what formal context is. Hence, to begin this review, it is necessary to introduce a unifying view of concept-cognitive learning, data mining, and knowledge discovery, which will be expressed in detail.

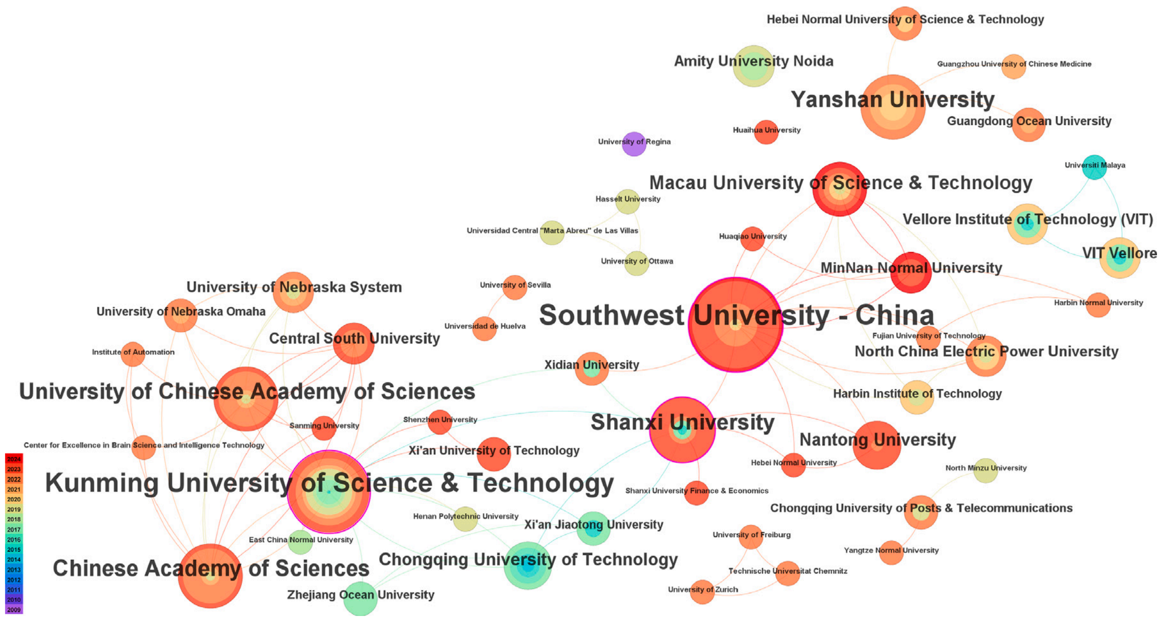

Fig. 5. Institutional collaboration network of CCL.

- Date set DS: a set that records descriptions about the object.
- Formal context F: the specific context obtained by processing the data set.
- Instance (or feature vector) x: a description of the object x.
- Feature (or attribute) a: the properties of some aspect of the object.
- Label $\mathcal{K} = \{1, 2, \ldots, l\}$: the decision attribute, also called decision class.
- Feature value (or attribute value): the value of a feature (or attribute).
- Object set (or object space) $U = \{x_1, x_2, \ldots, x_n\}$: the space spanned by objects.
- Feature set (or feature space) $A = \{a_1, a_2, \ldots, a_n\}$: the space spanned by features (or attributes).

With respect to a data set DS, we can build various formal contexts based on the different data processing techniques, including classical formal context, fuzzy formal context, interval formal context, etc. Generally, a formal context is a triple $F = (U, A, I)$ or a quintuple $F = (U, A, I, D, J)$, called the formal context or decision formal context, and the following holds.

- $U = \{x_1, x_2, \ldots, x_n\}$ is a nonempty finite object set.
- $A = \{a_1, a_2, \ldots, a_m\}$ is a nonempty finite attribute set.
- I is a binary relation on $U \times A$, e.g. $(x, a) \in I$ represents the object x has the attribute a.
- $U/D = \{D_1, D_2, \ldots, D_l\}$ is a decision division based on decision label D, where $D = D_1 \cup D_2 \cup, \ldots, \cup D_l$.
- $J : U \times D \to \{D_1, D_2, \ldots, D_l\}$ is a binary relation on $U \times D$.

In source data set and formal context, the acquisition of formal context is usually defined in terms of a specific data processing techniques, including normalization, discretization, regularization, fuzzification, etc. With respect to a formal context, we can formally define the cognitive learning operator of concept. Let $P(U)$ and $P(A)$ be two power sets of U and A, respectively. $\mathcal{L} : P(U) \to P(A)$ and $\mathcal{H} : P(A) \to P(U)$ are considered as a pair of set-valued mappings, and they are abbreviated as $\mathcal{L}$ and $\mathcal{H}$, respectively.

Definition 1. Let $F = (U, A, I)$ be a formal context. For any $X_1, X_2 \subseteq U$ and $B \subseteq A$, a pair of set-valued mappings $\mathcal{L}$ and $\mathcal{H}$ are called a pair of cognitive learning operators, if the following properties hold:

(1) $X_1 \subseteq X_2 \Rightarrow \mathcal{L}(X_2) \subseteq \mathcal{L}(X_1)$;
(2) $\mathcal{L}(X_1) \cap \mathcal{L}(X_2) \subseteq \mathcal{L}(X_1 \cup X_2)$;
(3) $\mathcal{H}(B) = \{x \in U | B \subseteq \mathcal{L}(x)\}$.

Note that Definition 1 directly declares three properties to define a pair of cognitive learning operators. In fact, these correspond to three cognitive viewpoints as follows.

- item (1) describes commonalities in cognitive viewpoints, i.e., the more samples a concept denotes, the fewer features it connotes, and vice versa.
- item (2) explains the cognitive viewpoint that the perception of the whole is more than the integration of those of its parts.
- item (3) declares that whether or not the information is selected depends on how relevant it is at the time.

The above Definition 1 mainly reflects the commonality cognition of concept, which constitutes the inverse Galois connection. Similarly, the characteristic cognition of concepts can be reflected by constructing order-preserving Galois connections, and the above pair of cognitive learning operators are defined as follows.

Definition 2. Let $F = (U, A, I)$ be a formal context. For any $X \subseteq U$ and $B_1, B_2 \subseteq A$, a pair of set-valued mappings $\mathcal{L}$ and $\mathcal{H}$ are called a pair of cognitive learning operators, if the following properties hold:

(1) $B_1 \subseteq B_2 \Rightarrow \mathcal{H}(B_1) \subseteq \mathcal{H}(B_2)$;
(2) $\mathcal{H}(B_1 \cup B_2) \subseteq \mathcal{H}(B_1) \cup \mathcal{H}(B_2)$;
(3) $\mathcal{L}(X) = \{a \in A | \mathcal{H}(a) \subseteq X\}$.

Whether the connection is the inverse or order-preserving Galois connection, the (X, B) is called a concept or formal concept, if only if $\mathcal{L}(X) = B$ and $\mathcal{H}(B) = X$, where X is the extent (that is, an object set) and B is the intent (that is, an attribute set) of the concept (X, B). Generally speaking, with respect to a pair of cognitive learning operators, we can produce a concept learning mechanism based on a formal context. In concept-cognitive learning, it is assumed that the concept is acquired through a pair of cognitive learning operators that satisfy the above three properties in Definition 1. Different from formal concept analysis theory, the constraints of cognitive learning operators of concepts appear to be more relaxed, especially in item (2) of Definition 1.

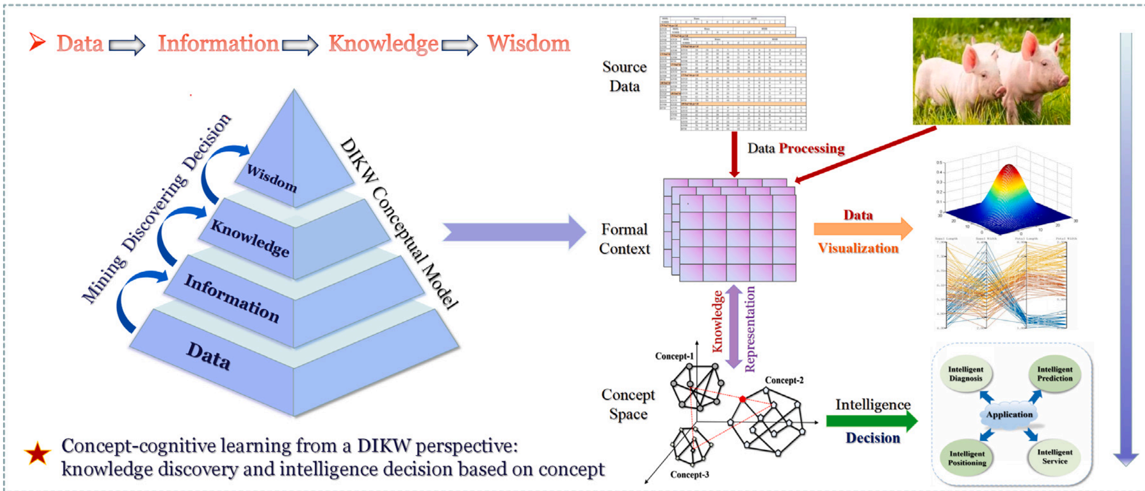

Fig. 6. Concept-cognitive learning: a DIKW perspective.

Definition 3. Let $F=(U,A,I)$ be a formal context, $\mathcal{L}$ and $\mathcal{H}$ be a pair of cognitive learning operators. For any $X\subseteq U$ and $B\subseteq A$, both $(\mathcal{HL}(X),\mathcal{L}(X))$ and $(\mathcal{H}(B),\mathcal{LH}(B))$ are concepts. Then, for any $x\in U$ and $a\in A$, we say that $(\mathcal{HL}(x),\mathcal{L}(x))$ and $(\mathcal{H}(a),\mathcal{LH}(a))$ are granular concepts

Hence, given a formal context $F=(U,A,I)$. For any concept (X,B), we have $\mathcal{L}(X)=\bigcap_{x\in X}\mathcal{L}(x)$, $\mathcal{H}(B)=\bigcap_{a\in B}\mathcal{H}(a)$. Then the following statements hold.

$(X,B)=\bigvee_{x\in X}(\mathcal{HL}(x),\mathcal{L}(x))=\bigwedge_{a\in B}(\mathcal{H}(a),\mathcal{LH}(a))$.

Furthermore, a dynamic concept-cognitive learning system for incremental learning can also be constructed according to the following definition.

Definition 4. Let U_{i-1} and U_i be object sets of $\{U_t\}\uparrow$ and A_{i-1} and A_i be attribute sets of $\{A_t\}\uparrow$, where $\{U_t\}\uparrow$ is a nondecreasing sequence subset of U, that is, $U_1\subseteq U_2\subseteq\cdots\subseteq U_n$; $\{A_t\}\uparrow$ is a nondecreasing sequence subset of A, that is, $A_1\subseteq A_2\subseteq\cdots\subseteq A_m$; Denote $\Delta U_{i-1}=U_i-U_{i-1}$ and $\Delta A_{i-1}=A_i-A_{i-1}$. Suppose

(1) $\mathcal{L}_{i-1}:2^{U_{i-1}}\to 2^{A_{i-1}}$, $\quad\mathcal{H}_{i-1}:2^{A_{i-1}}\to 2^{U_{i-1}}$,
(2) $\mathcal{L}_{\Delta U_{i-1}}:2^{\Delta U_{i-1}}\to 2^{A_{i-1}}$, $\quad\mathcal{H}_{\Delta U_{i-1}}:2^{A_{i-1}}\to 2^{\Delta U_{i-1}}$,
(3) $\mathcal{L}_{\Delta A_{i-1}}:2^{U_i}\to 2^{\Delta A_{i-1}}$, $\quad\mathcal{H}_{\Delta A_{i-1}}:2^{\Delta A_{i-1}}\to 2^{U_i}$,
(4) $\mathcal{L}_i:2^{U_i}\to 2^{A_i}$, $\quad\mathcal{H}_i:2^{A_i}\to 2^{U_i}$.

are four pairs of cognitive learning operators satisfying the following properties:

$$\mathcal{L}(x)=\begin{cases}\mathcal{L}_{i-1}(x)\cup\mathcal{L}_{\Delta A_{i-1}}(x), & if\ a\in A_{i-1}\\ \mathcal{L}_{\Delta U_{i-1}}(x)\cup\mathcal{L}_{\Delta A_{i-1}}(x), & otherwise\end{cases}$$

$$\mathcal{H}(a)=\begin{cases}\mathcal{H}_{i-1}(a)\cup\mathcal{H}_{\Delta U_{i-1}}(a), & if\ x\in U_{i-1}\\ \mathcal{H}_{\Delta A_{i-1}}(a), & otherwise\end{cases}$$

then we say that $\mathcal{L}_{i-1}$ and $\mathcal{H}_{i-1}$ are extended cognitive learning operators $\mathcal{L}_i$ and $\mathcal{H}_i$ with the update information $\mathcal{L}_{\Delta U_{i-1}}$, $\mathcal{H}_{\Delta U_{i-1}}$ and $\mathcal{L}_{\Delta A_{i-1}}$, $\mathcal{H}_{\Delta A_{i-1}}$.

Definition 5. Let $F=(U,A,I)$ be a formal context, $\mathcal{L}$ and $\mathcal{H}$ be two cognitive learning operators, $(\mathcal{HL}(x),\mathcal{L}(x))$ and $(\mathcal{H}(a),\mathcal{LH}(a))$ be two granular concepts. The granular concept space of U is a set of all granular concepts by $\mathcal{G}_{\mathcal{LH}}$, that is

$$\mathcal{G}_{\mathcal{LH}}=\{(\mathcal{HL}(x),\mathcal{L}(x))|x\in U\}\cup\{(\mathcal{H}(a),\mathcal{LH}(a))|a\in A\}.$$

where $\mathcal{G}_{\mathcal{LH}}$ is formed from granular concepts that can generally be identified by extent and intent.

Note that within both FCA and CCL, a formal context is used to represent the data to be analyzed and then extract concepts of different levels and the relationships between concepts from the formal context. FCA pays more attention to constructing a lattice structure of concepts, which are also called concept lattices. However, CCL emphasizes cognitive and learning concepts from a cognitive viewpoint, especially in the above cognitive learning operators, which also have cognitive properties. Furthermore, CCL can be used to research the cognitive and learning processes of things via these concepts. It is easy to find that the cognitive concepts learning from a formal context via a pair of cognitive learning operators are both structured knowledge and causal knowledge. Consequently, CCL is a theory and method with interpretability.

In concept-cognitive learning, it is evident that the formal context derived from the source data serves as the carrier of information, and the formal concept is selected as the carrier of knowledge. There are main reasons for this:

- Different scene requirements allow for the processing of source data into various forms of formal context. These forms enable the utilization of binary relationships to describe the association between objects and attributes.
- The involutivity between intent and extent of concepts effectively characterizes the knowledge structure and provides interpretability.
- The concept space constructed through formal concepts (obeying the Galois connection) provides a structured knowledge space suitable for storing knowledge.

Up to now, various concepts based on formal context have been proposed for the settlement of real practical problems. In particular, as shown in Fig. 6, knowledge discovery and intelligence decision analysis based on concept conform to the currently popular data-information-knowledge-wisdom (DIKW) hierarchy model, that is, data creates information, information creates knowledge, and knowledge creates wisdom. In this process, CCL places greater emphasis on transforming the source data into a specific formal context, representing the knowledge through concepts within this formal context, and ultimately utilizing these valuable concepts in the concept space to facilitate intelligent decisions by humans. In addition, it is worth noting that this hierarchy model also become an active research topic in artificial intelligence and data science.

4. Triadic categorization of CCL

As an emerging interdisciplinary research, the study of concept-cognitive learning is a topic covered in many disciplines, including philosophy, mathematics, cognitive science, computer science, and many

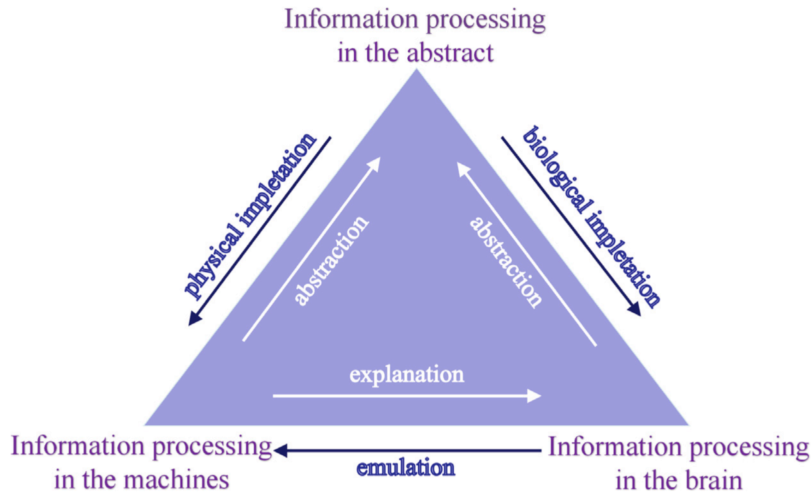

Fig. 7. The information processing triangle [15].

others. Namely, it is a highly inclusive research. As shown in Fig. 7, Yao [15] proposed the information processing triangle and provided value research thought, i.e., suggesting that cognitive concepts may be researched in three levels: in the abstract, in the machine, and in the brain. Consequently, how to examine CCL from the three levels of abstract-machines-brain will also be the guide of this section.

In developing a theory of three-way decision as thinking in threes, problem-solving in threes, and computing in threes, Yao [34] argued that a theory, a model, or a method with triadic structures are simple-to-understand, easy-to-remember, and practical-to-use. By following this philosophy of thinking in threes, we explore several triadic structures for characterizing CCL, namely, the information-processing triangle, the three research scopes (i.e., mathematics and logics, artificial intelligence, and cognitive simulations), the three research goals with respect to concept analysis methods, concept learning strategy, and concept cognitive mechanism, and three research methodologies (i.e., cognitive computing, granular computing, and machine learning).

4.1. Research scopes

Within the abstract-machines-brain three-level framework, this subsection will concentrate on CCL in the characterization and representation of concepts in mathematics and logics, the cognitive simulation in human-level or human-like brain logic, and the method design and application of artificial intelligence, i.e., the abstract level, the machines level, and the brain level.

4.1.1. Mathematics and logics

At the abstract level, CCL mainly focuses on the mathematics and logic of concepts, in which a core research content is concept formation and learning. As we all know, concepts are regarded as the most fundamental units of cognition in philosophy. Meanwhile, according to the standpoint of cognitive psychology and cognitive informatics, the concept is a knowledge structure existing in the human brain, and it is also a cognitive processing process carried out by the subject. It is often said that concepts are formed by abstracting the perceived essential characteristics of things and generalizing or induction in the process of cognition. Hence, describing the mathematical and logical structures of concepts is the primary concern of CCL at the abstract level.

In order to study the mathematical characterization of concepts, Wille [4] formally introduced the notion of formal concepts in 1982 to represent the knowledge structure of concept ontology within a formal context. This led to the development of formal concept analysis, a theory that utilizes order theory and complete lattices for data analysis, information processing, and knowledge management. Consequently, it has stimulated extensive exploration by researchers into the mathematical and logical structure underlying formal concepts. For instance, Ma et al. [35] propose a concept model based on a dual Galois connection, in which the zooming-in and zooming-out operators comprise two pairs of approximate operators. Wang et al. [36] investigated the logical and cognitive mechanism of the brain by utilizing cognitive informatics and formal methodologies. And then, Wang has investigated concept algebra [14] and conceptual knowledge structure [37] successively.

Additionally, Yao [15] identified the concept learning triangle, which consists of three sublevels for concept information processing: the philosophy level, the algorithm/technique level, and the application level. Building upon this research, various results on the structured description of concepts began to emerge, such as the mathematical model for concept systems [38], granule description based on FCA [39], attribute granulation in formal context [40], etc. Over time, numerous concepts have been proposed with specific meanings, such as abstract concepts [1], AFS-concept [41], three-way concept [42], concept tree [43], two-way concept [30], multi-adjoint concept [44] etc. These investigations have significantly contributed to the development of CCL.

4.1.2. Artificial intelligence

At the machine level, studies of the concept learning model and its application are typically the stress of CCL. Generally speaking, operating machines to learn knowledge from data has become a necessary link for the investigation of artificial intelligence, in which machine learning is a critical technology for discovering knowledge embedded in data [45,46]. Interestingly, the smallest unit of knowledge is a concept, and knowledge itself can be seen as a concept and even artificial intelligence is a concept. Therefore, at the machine level, concept learning has always been a necessary component for researching artificial intelligence. Meanwhile, the essence of concept learning lies in discovering and acquiring concepts, followed by conducting an in-depth analysis of the learned concepts, including analyzing the types and structures of concepts, relationships between concepts, and even using concepts to make decisions or guidance.

Early concept learning focuses on the study of a specific concept, such as Ref. [47] studied exemplar-based concept learning for heuristic classification, Ref. [48] offered a capturing conceptual factors method from multi-view data. In particular, a concept cognitive model based on granular computing, coined by Zhang and Xu in their seminal paper [13], greatly promoted the early development of cognitive concept learning. Subsequently, researchers gradually realized that the cognitive learning process of concepts can be explored through cognitive computing, enabling the simulation of cognitive learning mechanisms

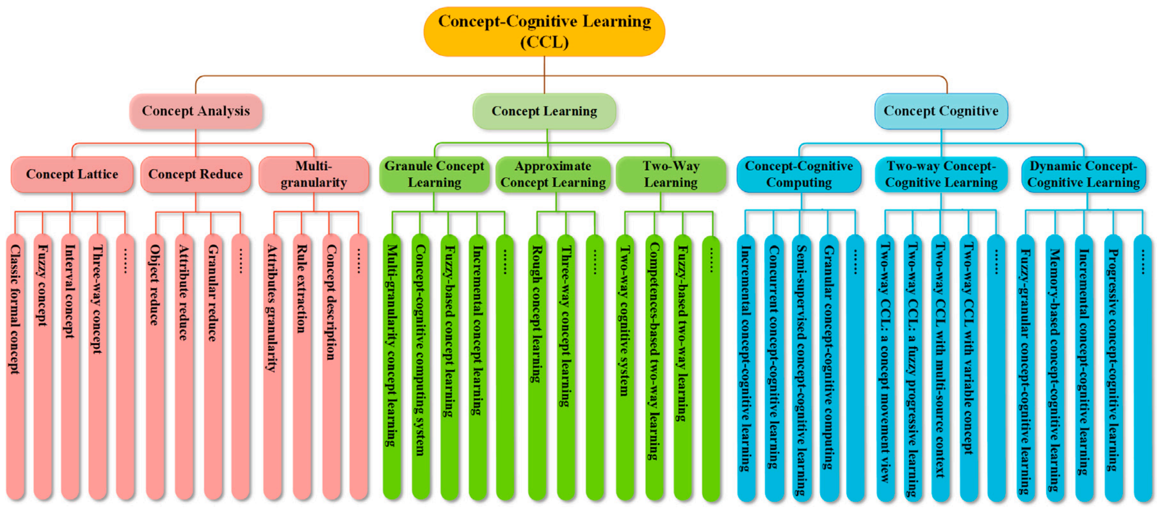

Fig. 8. Categorization of concept-cognitive learning [15].

observed in the human brain. However, these research studies have predominantly focused on the theoretical level: constructing concept learning systems, exploring various concept extensions, analyzing concept structures, analyzing knowledge reduction and rule extraction, and so on.

Undoubtedly, it remains challenging for a machine learning system to learn concepts with strong generalization performance and clear semantic interpretation, which has hindered progress in this field. Nevertheless, after a prolonged trough period, a new turn of concept learning at the machine level has benefited from the advancements of artificial intelligence technology, including granular computing, cognitive computing, and machine learning. Many valuable results began to emerge. For example, in the paper [17], a two-way concept learning mechanism was proposed to cognitive a sufficient and necessary information granular in a fuzzy formal context; Another paper [19] investigated a concept learning model that considered three cognitive properties to construct cognitive learning operators; Additionally, the paper [1] delved into the topic of human-level concept learning; Paper [29] presented a semi-supervised concept-cognitive learning approach based on concept space. Furthermore, the paper [49] explored the incremental learning mechanism of concept-cognitive learning for the classification task. These have significantly expanded the research horizon of concept-cognitive learning.

4.1.3. Cognitive simulation

At the brain level, CCL primarily focuses on concept learning by simulating human cognitive processes. In fact, various early explorations of concept learning (e.g., two-way learning [16], granular concept learning [19], concept learning system [38], etc.) and concept-cognitive learning belong to this category. It is worth mentioning that the distinction and connection between cognitive concept learning and concept-cognitive can be found in Ref. [31]. In addition, the current article is more concerned with concept-cognitive learning in a broad sense, including concept learning, cognitive concept learning, and concept-cognitive learning.

With the in-depth research of CCL in model, method, and application, increasing attention has been directed towards this research. For example, Kumar et al. [18] developed cognitive memory functionalities of bidirectional associative memory via concepts. Shivhare et al. [20] proposed a three-way conceptual approach for cognitive memory functionalities. Furthermore, Fan et al. [50] explored a multi-level cognitive concept learning strategy. Shi et al. [51] discussed an incremental concept learning method. Zhang et al. [43] investigated concept-cognitive learning using a concept tree. Liu et al. [52] developed an incremental incomplete concept-cognitive learning model. Additionally, Xu et al. introduced a two-way concept learning mechanism from a movement view [30] and a fuzzy progressive learning [31]. Guo et al. [2] combined the memory mechanism of humans with concept-cognitive learning to analyze fuzzy data and knowledge fusion, and many more. These achievements promote and enrich the in-depth study of concept-cognitive learning.

In conclusion, concept-cognitive learning can be studied from the three sublevels (i.e., abstract level, brain level, and machine level) of information processing triangle according to Fig. 7, and can also be regarded as a combination of the three.

4.2. Research goals

In this subsection, we present the tree diagram whose first two levels consist of three children, as shown in Fig. 8, to illustrate the categorization of concept-cognitive learning. The purpose of this diagram is to provide a concise overview of the concrete agenda of CCL within three categories: concept analysis method, concept learning strategy, and concept cognitive mechanism. Note that this diagram can be viewed as a list of some CCL models rather than a comprehensive summary.

4.2.1. Concept analysis method

The starting point of the category of concept analysis leverages concepts to characterize knowledge embedded in data. Then, it examines the properties of these concepts, the relations between them, and their applicability. Presently, the notable accomplishments of the concept analysis method of CCL concentrate on three aspects: concept lattice, concept reduction, and multi-granularity analysis.

Concept lattice [4] is a widely recognized approach for mining data associations and is a critical tool for data analysis and processing within FCA theory. Researchers have achieved numerous important breakthroughs, including various concept lattice constructions and generalizations. Initially, the focus of concept lattice research was Boolean data, with concepts being formalized by the 0–1 binary relationship between objects and attributes in the data, otherwise known as classical formal concepts. However, as research progressed, scholars realized that classical formal concepts represent only the simplest conceptual knowledge and that the demands of Boolean data for data collection and processing are often too strict. Consequently, some scholars have proposed various extension concepts, such as fuzzy concept [53], AFS concept [41], three-way concept [54,55], and others. Currently, concept lattices still serve as a necessary foundation for CCL research. However, current research in CCL focuses more on exploring the various concepts and the concept relationships in lattices rather than on how to construct lattices.

Furthermore, in order to optimize storage space and enhance knowledge discovery, concept reduction [56] is necessary. Concept reduction can be categorized into object reduction [57], attribute reduction [58], and granular reduction [59,60]. It is important to note that the goal of concept reduction is not to eliminate concepts entirely but rather to delete and utilize certain concepts based on specific requirements selectively. Additionally, there are two noteworthy studies: paper [2] introduces the partial concept forgetting for concept cognition, while paper [21] proposes a concept cognition strategy based on the big concept priority. In addition, the generalization of various concept models is inseparable from various multi-granularity analysis methods, including object granularity [61], attribute granularity [62], rule extraction [63], granular description [39], etc.

4.2.2. Concept learning strategy

The purpose of concept learning is to discover and acquire concepts and subsequently conduct in-depth analysis and application of the acquired concepts. Thus, the core research content of concept learning is the model and method used to discover and acquire concepts. For concept learning, numerous representative research about approximate concept learning [33,64], granule concept learning [19,21,65,66], and two-way learning [16,17,67,68] have also attracted wide attention.

Two-way learning is regarded as a prominent mathematical tool for concept learning that facilitates the acquisition of additional knowledge (i.e., diverse concepts) from the unknown via two learning methods: sufficient concept learning and necessary granule concept learning. Based on this naive thought of two-way learning, several related learning methods have been proposed to cater to different requirements. These include a two-way cognitive system for arbitrary information granule transformation [16], two-way concept learning for fuzzy formal context [17], interval-based two-way concept learning in interval-valued formal contexts [67], and so on. A two-way learning system is a cognitive process that learns from useless information. The process to cognitive the useless information into necessary, sufficient, sufficient and necessary information granules can be described as follows. Let $L_1 = P(U)$ and $L_2 = P(A)$ be two complete lattices, $\mathcal{L}$ and $\mathcal{H}$ be two cognitive learning operators (i.e., $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system). For any $X \in L_1$, $B \in L_2$, denote

$$\mathcal{G}_1 = \{(X, B) | B \leqslant \mathcal{L}(X), X \leqslant \mathcal{H}(B)\},$$
$$\mathcal{G}_2 = \{(X, B) | \mathcal{L}(X) \leqslant B, \mathcal{H}(B) \leqslant X\}.$$

- If $(X, B) \in \mathcal{G}_1$, then (X, B) is a necessary granule concept of $(L_1, L_2, \mathcal{L}, \mathcal{H})$. Meanwhile, $\mathcal{G}_1$ is a necessary granule concept space of $(L_1, L_2, \mathcal{L}, \mathcal{H})$;
- If $(X, B) \in \mathcal{G}_2$, then (X, B) is a sufficient granule concept of $(L_1, L_2, \mathcal{L}, \mathcal{H})$. Meanwhile, $\mathcal{G}_2$ is a sufficient granule concept space of $(L_1, L_2, \mathcal{L}, \mathcal{H})$;
- If $(X, B) \notin \mathcal{G}_1 \cap \mathcal{G}_2$, that is, (X, B) satisfy $B = \mathcal{L}(X)$ and $X = \mathcal{H}(B)$, then (X, B) is a sufficient and necessary granule concept of $(L_1, L_2, \mathcal{L}, \mathcal{H})$. Meanwhile, $\mathcal{G}_1 \cap \mathcal{G}_2$ is a sufficient and necessary granule concept space.

where $\leqslant$ is a quasi-order relationship.

In terms of granule concept learning, various models and methods have proposed innovative CCL using the object granule concept $(\mathcal{HL}(x), \mathcal{L}(x))$ or attribute granule concept $(\mathcal{H}(a), \mathcal{LH}(a))$. For instance, paper [33] introduced the idea of multi-granularity concept learning through the lens of three-way concepts from both positive and negative perspectives. Additionally, paper [27] developed a fuzzy-based concept learning based on conceptual clustering for efficient data analysis. To further enhance the efficiency of concept learning, paper [65] designed a concept cognitive computing system for dynamic classification.

Regarding approximate concept learning for CCL, two widely recognized learning methods have been studied: rough concept learning and three-way concept learning. For example, Li et al. [19] proposed a cognitive process for learning approximate cognitive concepts inspired by the approximate space of a rough set. Similarly, Guo et al. [64] focused on improving CCL accuracy using some approximate concepts. Furthermore, various three-way concept learning methods have been proposed from different perspectives. Huang et al. [66] considered an information fusion viewpoint, while Yuan et al. [49] suggested a progressive fuzzy three-way concept learning standpoint.

4.2.3. Concept cognitive mechanism

Concept cognitive is an active topic in artificial intelligence and cognitive science. As Lake in *Science* [1] pointed out, "People learning new concepts can often generalize successfully from just a single example, yet machine learning algorithms typically require tens or hundreds of examples to perform with similar accuracy". An emerging intelligence paradigm to concept-cognitive learning has opened up exciting new possibilities for advancing the field of concept cognition, i.e., mining and fusing knowledge from data. During the past few years, some novel concept-cognitive learning paradigms have begun to focus on studying the process of cognition and learning things via concepts that wish to enable machines to achieve human-level or human-like concept learning. A comprehensive overview of the developmental stages of CCL across different periods can be found in Ref. [31]. As mentioned in Ref. [31], since 2018, many scholars have increasingly emphasized the cognitive perspective in studying CCL. During this time, as shown in Fig. 1, the term "concept-cognitive learning" was adopted, marking the formal development stage of CCL. Currently, active research areas in CCL include cognitive computing, granular computing, and machine learning. For more detailed explanations of these fields, please refer to Section 4.3.

Regarding concept cognitive in CCL, researchers primarily carried out a series of studies in concept-cognitive computing, two-way concept-cognitive learning, and dynamic concept-cognitive learning. Note that the existing research about two-way learning [13,16] and two-way concept learning [17,67,69] has some issues leading to the stagnation of its related research, such as a complex learning mechanism and the absence of a concept evolution mechanism. To overcome these issues, based on the two-way learning and FCA theory, Guo and Xu [28,30,31] took the two-way cognitive process of sufficiency and necessity discrimination between objects and attributes as the starting point, studied the two-way cognitive mechanism of concept learning, and formally proposed the two-way concept-cognitive learning (TCCL). Paper [30] first investigated TCCL from the perspective of concept movement, in which authors also reveal the relationship between various two-way granular concepts (i.e., sufficient granular concept, necessary granular concept, sufficient and necessary granular concept), as shown in Fig. 9. Moreover, unlike two-way learning and two-way concept [13,16] learning [17,67,69], authors in paper [30] gave six concept cognitive methods for directly learning sufficient and necessary granule concepts as follows.

Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system, $\mathcal{G}_1 \cap \mathcal{G}_2 = \{(\mathcal{HL}(\mathcal{X}^{\mathcal{L}}), \mathcal{L}(X^{\mathcal{L}})) | X \in L_1\} \cup \{(\mathcal{H}(B^{\mathcal{H}}), \mathcal{LH}(B^{\mathcal{H}})) | B \in L_2\}$ be a sufficient and necessary fuzzy granule concept space. Then the following statements hold.

- $(\mathcal{HL}(X), \mathcal{L}(X)) \in \mathcal{G}_1 \cap \mathcal{G}_2$,
- $(\mathcal{HL}(X \wedge \mathcal{H}B)), \mathcal{L}(X \wedge \mathcal{H}(B)) \in \mathcal{G}_1 \cap \mathcal{G}_2$,
- $(\mathcal{HL}(X \vee \mathcal{H}(B)), \mathcal{L}(X \vee \mathcal{H}(B))) \in \mathcal{G}_1 \cap \mathcal{G}_2$,
- $(\mathcal{H}(B), \mathcal{LH}(B)) \in \mathcal{G}_1 \cap \mathcal{G}_2$,
- $(\mathcal{H}(B \vee \mathcal{L}(X)), \mathcal{LH}(B \vee \mathcal{L}(X))) \in \mathcal{G}_1 \cap \mathcal{G}_2$,
- $(\mathcal{H}(B \wedge \mathcal{L}(X)), \mathcal{LH}(B \wedge \mathcal{L}(X))) \in \mathcal{G}_1 \cap \mathcal{G}_2$.

Drawing inspiration from [30], paper [31] put forward the two-way concept-cognitive learning via fuzzy progressive learning for dynamic data updating. Then, the authors provide a convenient and innovative tool for researching CCL methods involving information fusion in the paper [28] to explore the two-way concept-cognitive learning within a multi-source formal context. Additionally, paper [70] analyzed the

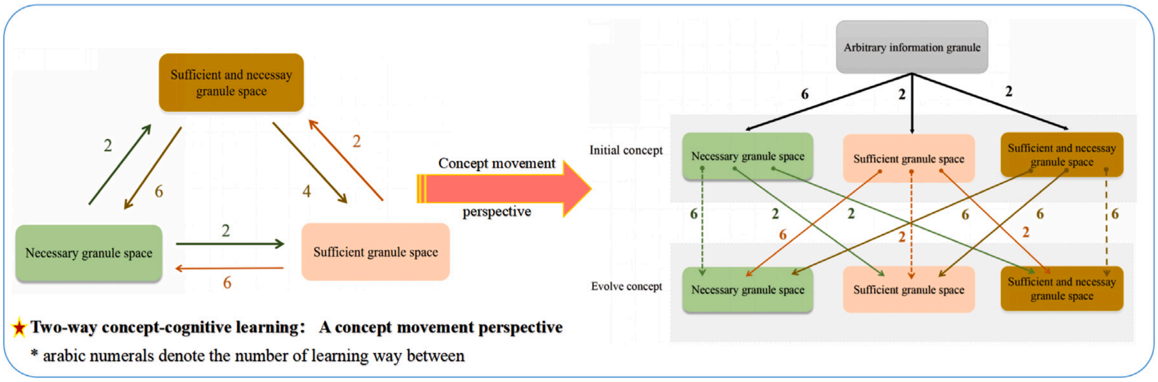

Fig. 9. Two-way concept-cognitive learning: a concept movement perspective [30].

dynamic updating mechanism for three-way concepts based on two-way concept-cognitive learning, further enriching the understanding of this area of study.

In concept-cognitive computing, Shi et al. [71] established a granular concept-cognitive computing system to extend the incremental learning of CCL and also proposed concurrent concept-cognitive learning for classification. Additionally, Mi et al. [29] combined some machine learning methods with CCL to study semi-supervised concept-cognitive learning. Furthermore, the dynamic concept-cognitive mechanism is a hot topic in concept-cognitive learning, including fuzzy-granular concept-cognitive learning [21,52], memory-based concept-cognitive learning [2], incremental concept-cognitive learning [51,72,73], incremental concept tree [43,74], and progressive concept-cognitive learning [31], etc. Meanwhile, a detailed introduction to them can be found in the related articles.

4.3. Research methodologies

As previously mentioned, CCL has been widely explored from different professional perspectives in various fields. Hence, this subsection mainly discusses the research methodologies of CCL from the perspectives of cognitive computing, granular computing, and machine learning.

4.3.1. Cognitive computing

Cognitive computing, a computer system that simulates the cognitive process of the human brain, is one of the fundamental technical fields within cognitive science [75,76]. Numerous studies have been conducted to explore this field from various professional perspectives. Note that the basic idea of CCL is to reveal the systematic law of the human brain through concept formation and learning. Therefore, integrating cognitive computing and concept learning is a nice choice for concept-cognitive learning. In this sense, various early explorations of concept learning, as well as current research on CCL, all involve the study of cognitive computing.

Numerous efforts have been dedicated to enriching the field of CCL by incorporating research ideas from cognitive computing, and its basic research contents include the cognitive concept mechanism, cognitive system construction, simulated cognitive agent behavior, decision analysis, etc. Specific cognitive mechanisms for concept-cognitive learning include incremental learning [49], fuzzy mechanism [5,27], memory mechanism [2], two-way learning [30,69], multi-level cognitive [50], etc. From these results, we can note that the theory and method of cognitive computing are very effective ways to study some problems in concept-cognitive learning, such as concept analysis, concept learning, and concept cognition. Thus, cognitive computing can be considered a significant approach for studying concept-cognitive learning, while the study of conceptual cognitive learning serves as an important complement to cognitive computing.

In fact, in cognitive psychology, there are four cognitive views regarding the understanding of concepts: the classical view, the prototype view, the exemplar view, and the theory-based view. Drawing inspiration from these views, Table 2 outlines the representative achievements of concept-cognitive learning.

4.3.2. Granular computing

Granular computing (GrC) [80–83] is influential studies for studying knowledge processing and concept learning, particularly in exploring how cognitive and learning a thing, one be viewed as a novel, interesting, and interpretable theory and technology. The current outline of GrC comprises formal concept analysis theory [84–86], fuzzy set theory [87–89], rough set theory [90–93], three-way decision theory [94–96], interval set theory [97–99], among others. Simultaneously, combining these theories with CCL has brought a lot of new problems, ideas, and methods. Table 3 summarizes some representative concepts and their cognitive learning operator of concept-cognitive learning under a GrC viewpoint.

Early attempts were made to combine fuzzy set theory with CCL (i.e., fuzzy-based CCL) in a fuzzy formal context due to two main advantages: (1) continuous data can be processed directly, avoiding information loss during the discretization process; (2) it can effectively handle the situation where the connotation of concepts cannot be accurately described when the intent of concept can be effectively measured. Furthermore, various fuzzy-based CCL models have been proposed for different problem scenarios. For instance, authors in papers [28,31] investigate the two-way CCL from a fuzzy-based progressive learning viewpoint and a multi-source information context. The authors in papers [49,73] proposed CCL based on fuzzy concept or weighted fuzzy concept for concept classification. Additionally, the authors in papers [2,5,21] discussed a fuzzy three-way concepts grounded on different information granularities for concept prediction, etc.

Note that three-way decision based on three-level thinking is another widely acknowledged theory of granular computing used for researching various concept models [100,101]. Inspired by this theory, authors [42,54] suggested the three-way concept by combining three-way decisions with the formal concept to study formal concept analysis from positive and negative perspectives (i.e., positive and negative attributes). Accordingly, we can also integrated the idea of three-way decisions or three-way concepts into CCL to study three-way concept-cognitive learning. For instance, paper [1] introduced three-way concept analysis into concept description, paper [49] discussed the incremental learning mechanism of the three-way concept, and paper [66] proposed a three-way concept learning method from an information fusion perspective. Additionally, it is worth mentioning the novel fuzzy-granular concept-cognitive learning by three-way decision studied in the paper [21]. This research clearly highlights that fuzzy three-way concepts have advantages over regular fuzzy concepts in terms of knowledge depiction and reducing cognitive bias.

Table 2
Cognitive views of CCL.

Cognitive views	Representative method	Research content	Reference
Classical view	Two-way concept learning	The cognitive process based on sufficient and necessary concept granule transformation.	[16,17,28,30,31,67,69,70]
Prototype view	Approximation concept learning	The cognitive process based on sufficient and necessary concept granule approximation.	[19,26,33,64,66,72,77]
Exemplar view	Concept classification learning	The classification learning process based on concept similarity degree.	[5,21,27,29,49,52,65,73]
Theory-based view	Human-level and human-like concept learning	The concept learning process by simulating the behavior of cognitive agents.	[2,13,19,31,43,50,74,78,79]

Table 3
Granular computing methods of CCL.

GrC model	Representative concepts	Cognitive learning operator	Reference
FCA	Classical formal concept (X, B)	$\mathcal{L}(X) = \{a \mid a \in A, \forall x \in X, (x, a) \in I\}$; $\mathcal{H}(B) = \{x \mid x \in U, \forall a \in B, (x, a) \in I\}$.	[16,30,52,78]
Rough set	Approximation concept of object set $(\underline{Apr}(X), \mathcal{L}(\underline{Apr}(X)))$, $(\overline{Apr}(X), \mathcal{L}(\overline{Apr}(X)))$	$\mathcal{L}$ and $\mathcal{H}$ satisfy definition 1 or definition 2	[19,64]
	Approximation concept of attribute set $(\mathcal{H}(\underline{Apr}(B)), \underline{Apr}(B))$, $(\mathcal{H}(\overline{Apr}(B)), \overline{Apr}(B))$	$\mathcal{L}$ and $\mathcal{H}$ satisfy definition 1 or definition 2	[19,64,93]
Fuzzy set	Fuzzy concept (X, B)	$\mathcal{L}(X) = \bigwedge_{x \in X} I(x, a), a \in A$; $\mathcal{H}(B) = \{x \in U \mid \forall a \in A, B(a) \leqslant I(x, a)\}$.	[17,27,28,31,69,70,79]
	Weighted fuzzy concept (X, B, w)	$\mathcal{L}(X) = \bigwedge_{x \in X} I(x, a), a \in A$; $\mathcal{H}(B) = \{x \in U \mid \forall a \in A, B(a) \leqslant I(x, a)\}$.	[73]
Three-way decision	Object-induced three-way concept $(X, (B_1, B_2))$	$\mathcal{L}(X) = (B_1, B_2)$; $\mathcal{H}(B_1, B_2) = X$	[5,49]
	Attribute-induced three-way concept $((X_1, X_2), B)$	$\mathcal{L}(X_1, X_2) = B$; $\mathcal{H}(B) = (X_1, X_2)$	[49]
	Fuzzy three-way concept $(\mathcal{H}^{\nabla}\mathcal{L}^{\nabla}(X), \mathcal{L}^{\nabla}(X))$	$\mathcal{L}^{\nabla}(X) = (\mathcal{L}(X), \mathcal{L}^{-}(X))$; $\mathcal{H}^{\nabla}(\mathcal{L}(X), \mathcal{L}^{-}(X)) = \mathcal{H}(\mathcal{L}(X)) \cap \mathcal{H}^{-}(\mathcal{L}^{-}(X))$.	[2,5,49]
	Fuzzy-granular three-way concept $(\mathcal{H}^{\nabla}\mathcal{L}^{\nabla}(x), \mathcal{L}^{\nabla}(x))$	$\mathcal{L}^{\nabla}(x) = (\mathcal{L}(x), \mathcal{L}^{-}(x))$; $\mathcal{H}^{\nabla}(\mathcal{L}(x), \mathcal{L}^{-}(x)) = \mathcal{H}\mathcal{L}(x) \cap \mathcal{H}^{-}\mathcal{L}^{-}(x)$.	[21]
Interval set	Interval-valued concept $(X, (B^L, B^U))$	$\mathcal{L}(X) = \{< a, [ma^L(X), Ma^U(X)] > \mid a \in A\}$; $\mathcal{H}(B^L, B^U) = \{x \mid x \in U, \forall a \in B, a^L(X) \geq B^L(a), a^U(X) \leq B^U(a)\}$.	[67]

In addition, other models of granular computing, such as rough sets and interval sets, have also been integrated into the study of CCL. As a result, several related CCL models have been proposed successively, including cognitive learning of approximate concepts [77], interval-value concept-cognitive learning [67], etc. Various common granulation methods of GrC have also been applied to the cognitive concepts for concept-cognitive learning systems, such as attribute granulation [50], object granulation [102], relation granulation [61,103], and so on.

4.3.3. Machine learning

From a machine learning perspective, concept-cognitive learning has both cognitive properties and the capability to accomplish two crucial learning tasks: classification and prediction. An integration of these two aspects expands the research horizon in this field. Fig. 10 illustrates the comprehensive procedure of concept-cognitive learning from a machine learning standpoint. This procedure encompasses three stages: concept learning, concept cognition, and concept recognition. The specific details are outlined below:

- Concept learning: this stage involves data processing for the specific formal context and learning concepts in the formal context.
- Concept cognitive: the concept formation via a pair of concept cognitive learning operators and mapping these concepts to different concept subspaces.
- Concept recognition: the concept application for actual scenarios by concept recognition and knowledge discovery from the unknown.

Against this background, numerous CCL methods have been established for implementing the different required problem scenarios. For example, a multi-attention CCL for concept learning on the handwritten numeral [78], two-way CCL for multi-source information context [28], fuzzy-granular CCL for dynamic knowledge discovery [21], memory-based CCL for fuzzy data classification and knowledge fusion [2], multi-level CCL to recognize and distinguish micro-expressions [79], semi-supervised CCL for object classification task [29], fuzzy-based CCL to tumor diagnosis analysis [5], CCL based on conceptual clustering to exploit knowledge from fuzzy data [27], a stochastic strategy based CCL for incremental incomplete concept learning [52], a novel CCL method for bird song classification [22], incremental CCL model for concept classification oriented to weighted fuzzy concepts [73], etc. These results have significantly enriched and advanced CCL research.

5. Challenges and future directions

CCL provides a multi-view, multi-level, and multi-granularity intelligence computing paradigm for the description and problem-solving of complex problems and has attracted the attention of many researchers. Although concept-cognitive learning has shown some satisfactory work in research scopes, goals, and methodologies, it is essential to acknowledge a range of limitations of the current study thus far. For

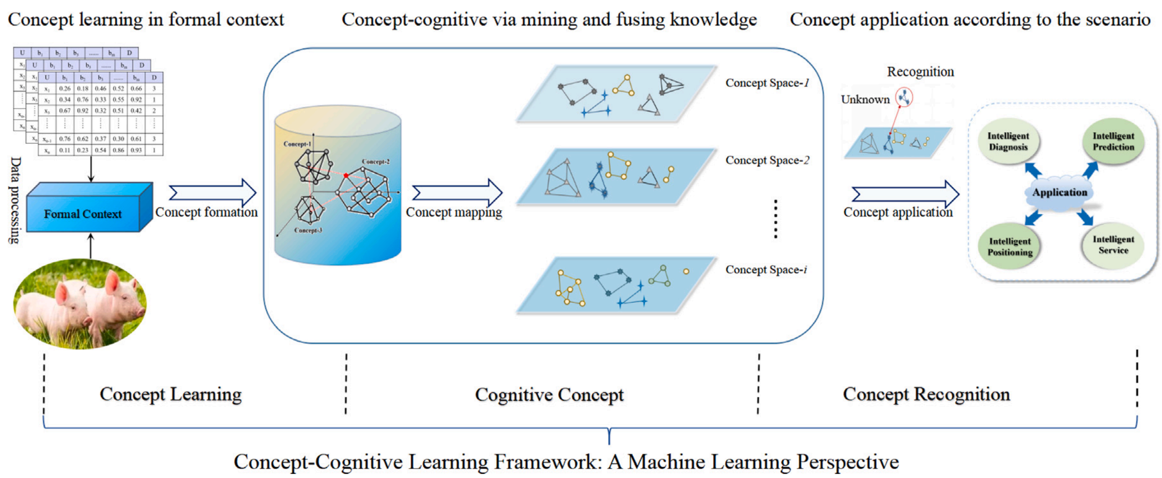

Fig. 10. Concept-cognitive learning: a machine learning perspective.

example, CCL has not been applied to complex problem-solving scenarios, including cognitive and learning in big data environments and multi-modal scenarios. Moreover, as an emerging field, CCL is gradually becoming recognized and hot in artificial intelligence and cognitive computing. However, CCL research is still in its infancy. There is a big scope for exploring alternative scopes, goals, and methodologies of concept-cognitive learning. Therefore, a future expectation of the model, method, and application of CCL in this section will be discussed from six aspects: concept learning method, concept cognition mechanism, cognitive system construction and optimization, complex decision-making, interdisciplinary research, and engineering applications.

- Concept learning method: Investigating human-level or human-like concept learning methods is the first concern for concept-cognitive learning. Note that human-level concept learning [1] needs one or a few instants, yet a central challenge is how to succeed in learning concepts from such small sample data. For most machine learning and concept learning methods, fitting a more complicated model requires more data, not less. In addition, continuous learning from non-stationary data streams is an intriguing research area. Current studies on concept learning typically assume that important elements of the learning process remain unchanged within a closed environment. However, in reality, data accumulates over time, making it challenging to train a concept learning system using traditional approaches that rely on collecting all the data upfront. Consequently, studying concept-cognitive learning in a small sample and non-stationary data environment holds significant importance in achieving human-level or human-like concept learning.
- Concept cognition mechanism: One of the key research areas in CCL is the exploration of cognition mechanisms in diverse and complex scenarios. The current concept cognition system consists of cognitive operators, specifically extent–intent cognitive learning operators, and intent–extent operators, which process data into concepts. To tackle more complex data analysis tasks, CCL needs to integrate the cognitive operator of the concept cognitive system with specific real-world data environments. For example, in the multi-modal background, one concept may be expressed in various forms, such as images, text, and audio. How does CCL study the representation and characterization of this concept? Hence, CCL research also needs to deal with some challenges, including processing complex data like incomplete, mixed, heterogeneous, and multi-modal data, as well as adopting new learning methods like parallel learning, distributed learning, multi-modal learning, etc.
- Cognitive system construction and optimization: To tackle more complex learning tasks, we need to focus not only on enhancing the effectiveness of the concept-cognitive learning system but also on ensuring its stability. Most existing concept-cognitive learning systems are constructed using classical cognitive learning operators, where a basic assumption is that cognitive learning operators have the function of complete cognition. However, in reality, affected by various uncertain factors (e.g., missing data, fuzzy data, insufficient cognition, cognitive dimension uncertainty, and others), it is often impossible to realize the complete cognition of concepts, which is manifested in the deviation of cognitive results of the cognitive system. In addition, integrating language understanding of concepts with other modalities, such as vision and audio, can lead to more robust cognitive systems capable of understanding and generating content across multiple modalities. Hence, exploring the effective cognitive learning operator is a valuable issue for CCL. Furthermore, evaluating the effectiveness and stability of the system warrants further investigation.
- Complex decision-making: Intelligent decision-making is a crucial objective of data science. Discovering valuable information and knowledge from data to enable intelligent decision-making by humans is also a critical topic in CCL. However, the openness of the decision environment, virtualization of decision resources, unstructured decision problems, and collaborative problem-solving increase the difficulty of solving decision problems in big data environments, especially for complex decision problems such as major engineering decisions, large enterprise management decisions, and social public event processing decisions. The current concept-based intelligent decision-making struggles to perform well when dealing with the multi-modal, spatiotemporal dynamics, and multi-source heterogeneity of knowledge discovery in big data. As we all know, cognitive computing plays an important role in big data intelligence mining, but how to build a specific cognitive model for big data intelligence decision-making is still a challenging subject. As such, how to simulate the human cognitive learning process and construct specific concept-cognitive learning models that advance the development of intelligent decision-making is also an open question.
- Interdisciplinary research: Indeed, concept-cognitive learning is an interdisciplinary emerging research. Exploring the integration of CCL with emerging disciplines in recent years is a topic worthy of investigation. As mentioned in Section 4.3, some progress has been made in the cross-research of CCL and granular computing, cognitive computing, and machine learning. Nevertheless, some areas, such as large model construction based on CCL, space–time concept cognitive learning, big data concept-cognitive learning, multi-modal concept-cognitive learning, and others, present

promising avenues for exploration. In addition, as an important research topic of artificial intelligence and cognitive computing, CCL can be combined with many branches of data science to get some new research topics on artificial intelligence. For example, one can combine with a graph network to produce concept-cognitive learning based on a graph network structure and also can study concept-cognitive learning with deep learning so as to complete many artificial intelligence tasks, such as image recognition and natural language processing. Therefore, it will be helpful to further improve the theory research of concept-cognitive learning by drawing on the relevant theories of concept representation and learning in interdisciplinary research.

- Engineering applications: Engineering application plays a crucial role in showcasing research outcomes and translating them into practical benefits. However, it is indeed one of the weaknesses of CCL research. Meanwhile, in the era of digital industrial transformation with the internet of everything (IoE), traditional industrial production methods have been unable to satisfy the demands of intelligent society for intelligent manufacturing. While CCL has achieved some progress in fields like medical diagnosis, machine learning, and data mining, it has predominantly focused on theoretical research and lacks exploration of engineering application scenarios. Drawing on many artificial intelligence technologies, carrying out advanced CCL research, actively excavating the intelligent application of CCL in various industries is the key to the subsequent participation in engineering applications, such as smart cities, smart parks, smart transportation, smart medical care, smart education, and other intelligence scenarios. Consequently, the concept-cognitive learning for engineering application emerges will be a critical point that demands concentration in the future.

6. Conclusion

Concept-cognitive learning, a crucial technology for knowledge discovery and representation, has been gaining acceptance and recognition. One immense potential of CCL is promoted by the various formal concept constructs by the cognitive learning operators as the knowledge carrier for knowledge discovery and intelligence decisions. In fact, the research objective of CCL is to simulate the cognitive learning mechanism of humans, which can also be regarded as a research category for the intelligence exploration of humans or animals. This survey aims to provide an overview of research articles on CCL, summarizing numerous state-of-the-art concept-cognitive learning techniques.

Up to now, we have witnessed a growing interest and the growth of CCL from the view of the information-processing triangle (i.e., the abstract-machine-brain level). To advance concept-cognitive learning, constructing an emerging intelligence paradigm that mines and fuses knowledge from data is crucial. This survey provides the first comprehensive review of CCL, including its hierarchical classification, representative model, theoretical significance, application value, and future development potential. This survey aims to promote the development of CCL and attract increasing attention to CCL.

CRediT authorship contribution statement

Doudou Guo: Writing – original draft, Methodology, Conceptualization. **Weihua Xu:** Writing – review & editing, Methodology, Investigation. **Weiping Ding:** Supervision, Methodology, Conceptualization. **Yiyu Yao:** Methodology, Formal analysis, Conceptualization. **Xizhao Wang:** Writing – review & editing, Methodology, Investigation. **Witold Pedrycz:** Writing – review & editing, Methodology. **Yuhua Qian:** Writing – review & editing, Supervision, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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