Knowledge-Based Systems 71 (2014) 114–125

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

QMIQPN: An enhanced QPN based on qualitative mutual information for reducing ambiguity

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A R T I C L E I N F O

Article history:
Received 4 February 2013
Received in revised form 1 June 2014
Accepted 23 July 2014
Available online 30 July 2014

Keywords:
Knowledge representation
Qualitative reasoning
Qualitative Probabilistic Networks
Qualitative mutual information
Ambiguity reduction

A B S T R A C T

Enhanced Qualitative Probabilistic Network (QPN) is to make qualitative network more applicable by reducing ambiguity in a qualitative way. To reduce ambiguity in the basic QPN inference, we propose an enhanced QPN based on qualitative mutual information (QMI), named QMIQPN. Firstly, we give a strict definition of QMI. Secondly, based on the definition, we present the formalism of QMIQPN. Specifically, we take QMI as the strength of qualitative influence in QMIQPN, the qualitative influence with strength differs from the previous work, which additional expressiveness of the enhancement does not come at the expense of the property of symmetry of influence. Thirdly, we analyze several relative properties of qualitative influences with strengths. Furthermore, we improve the *Sign-propagation Algorithm* to reduce ambiguity and discuss its complexity. Finally, by experiments on several databases, we analyze the performance of QMIQPN. Theoretic analysis and experimental results illustrate that QMIQPN is qualitative and efficient, yet allows for reducing some ambiguities upon QPN inference.

1. Introduction

Qualitative Probabilistic Network (QPN) [1] occupies an important region in qualitative representation space because of incomplete knowledge and unnecessarily strictly numeric precise for many applications, and it becomes a popular frame for knowledge representation in artificial intelligence as Bayesian network [2], Semantic model [3], Conceptual Graph and Resource Description Framework [4], etc. Furthermore, reasoning with qualitative probabilities is much more efficient than reasoning with precise ones, the inference complexity of QPN is a polynomial in the size of the network [5], rather than NP-hard [6]. Therefore, many approaches have been proposed for QPN modeling and inference according to various kinds of applications [7–20].

However, in a QPN, influence relationships between variables can be modeled as the direction of the shift in the distribution, but no indication of their strengths can be provided as in a quantified network. The major drawback of this coarse level is that ambiguous signs easily arise upon inference, moreover, once an ambiguous sign has arisen, it will spread throughout major parts of the network. Although not incorrect, ambiguous signs provide no information and are not very useful in practice [14]. Thus ambiguity reduction, also called trade-off resolution, is a common problem in the basic QPN inference.

Recently, many methods for reducing ambiguity have been developed. Parsons [8] has introduced the concept of categorical influence, which is either an influence that increases a probability to 1, or an influence that decreases a probability to 0, and thus serves to reduce some ambiguities in which it is involved. Liu and Wellman [21] have designed two methods for reducing ambiguities based upon the idea of reverting to numerical probabilities whenever necessary, the methods require the fully quantified probabilistic network. Renooij et al. [22] have also investigated the use of order-of-magnitude *kappa* values to capture influence strengths in a QPN and detailed the use of these kappas upon inference, thereby providing for ambiguity reduction. Renooij et al. [14] have also presented an enhanced formalism of QPN (EQPN) which introduces a notion of relative strength by distinguishing between weak and strong influences for ambiguity reduction. Renooij et al. [9] have presented an algorithm for dealing with unresolved trade-offs that builds upon the idea of zooming in on the part of a QPN where the actual trade-offs reside. It is a different approach to dealing with trade-offs, the authors have proposed to isolate the unresolved trade-offs and identify from the network the

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http://dx.doi.org/10.1016/j.knosys.2014.07.014
0950-7051/

information that would serve to resolve them, rather than resolving them by providing an even finer level of detail. Yue et al. [18] have adopted the probabilistic-rough-set-based weights to the qualitative influence, and have presented an enhanced QPN (EQPN) of providing for a finer level of representation detail. Yue and Liu [19] have taken another EQPN with interval probability parameters as indicators of influence strengths to reduce ambiguity.

Unfortunately, the methods [14,18,19] for reducing ambiguity by providing the strengths of influence relationships have some disadvantages, they are as follows:

(1) Some additional expressiveness of their enhancement come at the expense of the property of symmetry of influences, which may bring about higher computation complexity.
(2) The combining signs of parallel influences are no longer associative, which can result in loss of information upon combining several trails having strong and weak conflicting influences. To achieve the more informative and unique result, the order of nodes combination is elicited before combining them. However, such heuristics could increase the complexity.
(3) The time complexity of inference in these QPNs will be increased by using *Sign-propagation Algorithm*, and perhaps become exponential.

Thus the main motivation of this paper is to avoid the above disadvantages as much as possible in our method.

It is well known that probability-based information theory has been founded by Shannon [23,24], in recent years, information-theoretic measures are increasingly used in many researches [25–27]. For example, the mutual information(MI) is a measure of the strength of association between variables and it exhibits the property of symmetry. To handle the asymmetry of an influence's strength in [14,18,19] and to reduce ambiguity in a QPN, we will propose an enhanced formalism of QPN (QMIQPN) and give some strict definitions, the details of relative properties in a QMIQPN, complexity analysis, experiments, etc.

The rest of this paper is organized as follows. Some basic concepts about QPN are briefly reviewed in Section 2. In Section 3, we give a strict definition of QMI and present a new formalism of enhanced QPN (QMIQPN) based on the definition, and then prove four relative properties of qualitative influence in a QMIQPN. Section 4 improves the basic *Sign-propagation Algorithm* to reduce ambiguity and discusses its complexity. Section 5 analyzes the performance of QMIQPN by simulating experiments on several databases. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, we will review some basic concepts about QPN.

2.1. The Basic QPN

QPN [1] is proposed as the qualitative abstraction of probabilistic network or Bayesian network (BN) [2]. It encodes statistical variables and probabilistic relationships between them in a directed acyclic graph (DAG). The relationships between variables are not quantified by conditional probabilities as in a BN but are summarized by the signs of qualitative relationships instead. That is, a QPN $Q = (G, \delta)$ also comprises a DAG $G = (V, E)$ modeling variables and probabilistic relationships between them, and each *sign* δ ($\delta \in \{+, -, 0, ?\}$) of qualitative influence that indicates the shift direction between variables in the distribution. For abbreviation, all variables are assumed to be binary and their values are ordered, i.e., writing x for $X = True$ (or $X = 1$) and $\bar{x}$ for $X = False$ (or $X = 0$), and $x > \bar{x}$ (or $1 > 0$).

The qualitative relationships include qualitative influences and qualitative synergies, qualitative influences include positive influence, negative influence, zero influence and ambiguous influence. Since qualitative synergies are not used for reducing ambiguity in this paper, we will not discuss them, the interested reader can refer to [1,28].

Definition 1 (*Positive Influence [1,14]*). Let $Q = (G, \delta)$ be a QPN, $G = (V, E)$ be a DAG, and Pr be a joint probability distribution on V, and X, Y be variables, and $X \rightarrow Y \in E$, a positive influence of variable X on its successor Y, written $S^{+}(X, Y)$, iff

$$\Pr(y|xm) \geqslant \Pr(y|\bar{x}m) \quad (1)$$

for any combination of values m for the set $\pi(Y) \setminus \{X\}$ of predecessors of Y other than X.

The definition expresses the fact that observing a high value for X makes the higher value for Y more likely, regardless of any other direct influences on Y. A *negative influence*, denoted by S^{-}, and a *zero influence*, denoted by S^{0}, are defined analogously, just substitute $\leqslant$ and $=$ for $\geqslant$ in the formula (1), respectively. If the influence of X on Y is positive for one combination of m and negative for another combination, in other words, if it is non-monotonic or unknown, the influence is called *ambiguous influence*, denoted by $S^{?}$.

Now a piece of fictitious medical knowledge is described to serve as our example application throughout the paper.

Example 1 (*From [9,29]*). We consider the probabilistic *Radiotherapy* network shown in Fig. 1(a). Node T models the therapy instilled, R models a reduction of the tumor, and S models the development of scar tissue. L models the life-expectancy of a patient after therapy, where l indicates that the patient will survive for at least 6 weeks. Thus we can obtain the corresponding qualitative abstraction of *Radiotherapy* network and show its QPN as Fig. 1(b).

2.2. Sign-propagation Algorithm

Since variables not only influence each other directly along arcs, they can also exert indirect influence on one another. Three properties [1,14] hold for these qualitative influences, namely *symmetry*, *transitivity* and *composition* properties.

- The property of *symmetry* guarantees that, if a network includes the influence $S^{\delta}(A, B)$, then it also includes $S^{\delta}(B, A)$ with the same sign δ.

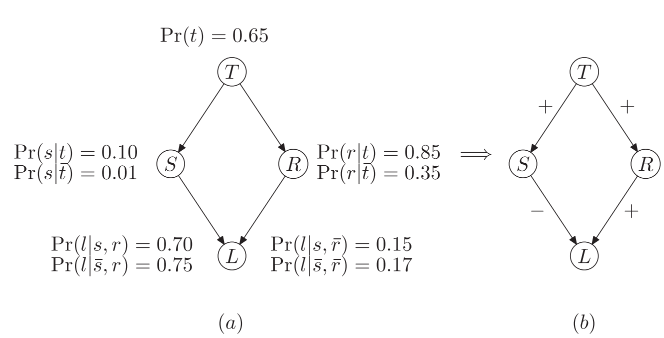

Fig. 1. An example of *Radiotherapy* QPN abstracted from BN. (a) *Radiotherapy* BN. (b) *Radiotherapy* QPN.

- The property of *transitivity* asserts that qualitative influences along an active trail without head-to-head nodes combine into an indirect influence whose sign is determined by the $\otimes$-operator from Table 1.
- The property of *composition* asserts that multiple qualitative influences between variables along parallel active trails combine into a composite influence whose sign is determined by the $\oplus$-operator from Table 1.

For inference in a QPN, Druzdzel and Henrion [5] have proposed an efficient algorithm termed *Sign-propagation Algorithm*. The goal of this algorithm is to determine the sign of each target node. In essence, the algorithm computes the influence sign along all active trails between the newly observed variable and the other variables in the network, building upon the properties of symmetry, transitivity and composition of influences. The algorithm is summarized in pseudocode as Algorithm 1.

Algorithm 1. Sign-propagation Algorithm in QPN

Input: A QPN, the evidence node (the observed node) and its sign $\delta \in \{+, -, 0, ?\}$
Output: δ of each target node
for each node $V_i \in V$ in QPN **do**
  $\delta[V_i] \leftarrow$ '0'
  PropagateSign($\emptyset, O, O, \delta$ *of evidence node*)
end for
Procedure PropagateSign(*trail*, *from*, *to*, *messagesign*):
$\delta[to] \leftarrow \delta[to] \oplus messagesign$
$trail \leftarrow trail \cup \{to\}$
for each active neighbor V_i of *to* **do**
  $linksign \leftarrow \delta$ of influence between *to* and V_i
  $messagesign \leftarrow \delta[to] \otimes linksign$
  if $\delta[V_i] \notin trail$ and $\delta[V_i] \neq \delta[V_i] \oplus messagesign$ **then**
    PropagateSign($trail, to, V_i, messagesign$)
  end if
end for

In this algorithm, the character of the sign addition operator implies that each node can change its sign at most twice — first from '0' to '+', '–', or '?' and then from '+' or '–' only to '?', which can never change to any other sign. From this observation, we have that no variable is ever visited more than twice upon inference, which guarantees that the algorithm can be halt and be quadratic in the size of the network.

3. QMIQPN

In this section, we first describe the problem of inference ambiguity in basic QPN by example, and then propose an enhanced QPN based on qualitative mutual information, named QMIQPN.

Table 1
The $\otimes$- and $\oplus$-operators.

$\otimes$	+	–	0	?	$\oplus$	+	–	0	?
+	+	–	0	?	+	+	?	+	?
–	–	+	0	?	–	?	–	–	?
0	0	0	0	0	0	+	–	0	?
?	?	?	0	?	?	?	?	?	?

3.1. The description of inference ambiguity problem

Example 2 (*Continued from Example 1*). If the patient receives radiotherapy, his life expectancy may decrease due to stenosis and will on the other hand increase if the tumor is reduced. From the probabilities in the quantitative network, we have that the effect of stenosis on life expectancy is much smaller than the effect of tumor reduction. That is, the influence of S on L is much weaker than the influence of R on L is.

However, in a qualitative network shown in Fig. 2(a), we only know node-sign. Suppose that a patient is taking radiotherapy. The variable T is observed and we update its node-sigh to '+'. According to Sign-propagation Algorithm, variable T thereupon propagates a message, with sign '$+ \otimes + = +$', towards S. Variable S updates its node-sign to '+' and sends a message with sign '$+ \otimes - = -$' to L. Variable L updates its node-sign to '$-$', it sends no messages as it has no neighbors that need to update their sign. Variable L does not pass on a sign to R, since the trail from S via L to R is not active. Variable T also sends a message on the other hand, with sign '$+ \otimes + = +$', to R. Variable R updates its node-sign accordingly and passes a message with sign '$+ \otimes + = +$' to L. Variable L thus receives the additional sign '+'. This sign is combined with the previously updated node-sign '$-$', which results in the ambiguous node-sign '$- \oplus + = ?$' for L, which shows in Fig. 2(b).

Note that in a QPN, such an ambiguity will arise when parallel influences with opposite basic signs are combined with the $\oplus$-operator in Table 1. In fact, to some extent, the ambiguity can be reduced. Thus we will study the problem of ambiguity reduction in this paper.

3.2. Qualitative mutual information

In general, we can measure the strength between random variables (also read "nodes") X and Y using mutual information [25].

Definition 2 (*Mutual Information, MI [30]*). Consider two random variables X and Y with a joint probability distribution $\Pr(x, y)$ and the marginal probability distribution $\Pr(x)$ and $\Pr(y)$. The mutual information (MI) $I(X;Y)$ is the relative entropy between the joint distribution and the product distribution $\Pr(x)\Pr(y)$, that is,

$$I(X;Y) = \sum_{Y_j \in Y} \sum_{X_i \in X} \Pr(X_i, Y_j) \log \left(\frac{\Pr(X_i, Y_j)}{\Pr(X_i)\Pr(Y_j)} \right), \qquad (2)$$

where $0 \leqslant I(X;Y) \leqslant 1$.

The MI between nodes can tell us if the two nodes are dependent and if so, how close their relationship is. The bigger the $I(X;Y)$ is, the closer the relationship of variable X and Y is. We would claim that X and Y are independent iff $I(X;Y) = 0$.

Moreover, we have the following theorems [30] on MI.

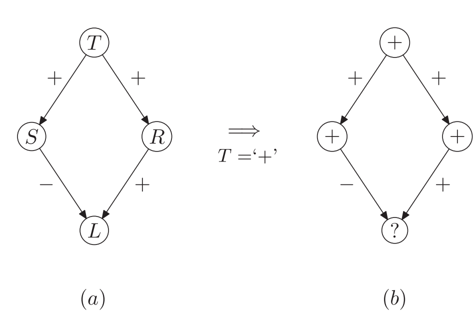

Fig. 2. The ambiguity in QPN inference. (a) *Radiotherapy* QPN. (b) Given T = '+', the ambiguity arises in node L.

Theorem 1 (*Symmetry*).

$$I(X;Y) = I(Y;X). \quad (3)$$

Theorem 2 (*Data-processing Inequality*). *If $X \to Y \to Z$ is a Markov chain, then*

$$I(X;Y) \geqslant I(X;Z) \text{ and } I(Y;Z) \geqslant I(X;Z). \quad (4)$$

Theorem 3 (*Chain Rule for Information*).

$$I(X_1, X_2, \ldots, X_n; Y) = \sum_{i=1}^{n} I(X_i; Y|X_{i-1}, X_{i-2}, \ldots, X_1). \quad (5)$$

In a QPN, Given edge (X, Y) or $X \to Y$, all variables are assumed to be binary, so the MI between X and Y is

$$I(X;Y) = \Pr(x,y)\log\left(\frac{\Pr(x,y)}{\Pr(x)\Pr(y)}\right) + \Pr(x,\bar{y})\log\left(\frac{\Pr(x,\bar{y})}{\Pr(x)\Pr(\bar{y})}\right) + \Pr(\bar{x},y)\log\left(\frac{\Pr(\bar{x},y)}{\Pr(\bar{x})\Pr(y)}\right) + \Pr(\bar{x},\bar{y}) \times \log\left(\frac{\Pr(\bar{x},\bar{y})}{\Pr(\bar{x})\Pr(\bar{y})}\right). \quad (6)$$

If variable X positively influences Y, that is, $S^+(X,Y)$. According to Definition 1, we have

$$\Pr(y|x) \geqslant \Pr(y|\bar{x}). \quad (7)$$

Further,

$$1 - \Pr(y|x) \leqslant 1 - \Pr(y|\bar{x}) \iff \Pr(\bar{y}|x) \leqslant \Pr(\bar{y}|\bar{x}). \quad (8)$$

and since

$$\Pr(y) = \Pr(y|x)\Pr(x) + \Pr(y|\bar{x})\Pr(\bar{x}). \quad (9)$$

$$\Pr(\bar{y}) = \Pr(\bar{y}|x)\Pr(x) + \Pr(\bar{y}|\bar{x})\Pr(\bar{x}). \quad (10)$$

the formula (7) and (8) are substituted into the formula (9) and (10), so

$$\Pr(y) = \Pr(y|x)\Pr(x) + \Pr(y|\bar{x})\Pr(\bar{x}) \leqslant \Pr(y|x)\Pr(x) + \Pr(y|x)(1 - \Pr(x)) = \Pr(y|x). \quad (11)$$

$$\Pr(y) = \Pr(y|x)\Pr(x) + \Pr(y|\bar{x})\Pr(\bar{x}) \geqslant \Pr(y|\bar{x})(1 - \Pr(\bar{x})) + \Pr(y|\bar{x})\Pr(\bar{x}) = \Pr(y|\bar{x}). \quad (12)$$

$$\Pr(\bar{y}) = \Pr(\bar{y}|x)\Pr(x) + \Pr(\bar{y}|\bar{x})\Pr(\bar{x}) \leqslant \Pr(\bar{y}|\bar{x})(1 - \Pr(\bar{x})) + \Pr(\bar{y}|\bar{x})\Pr(\bar{x}) = \Pr(\bar{y}|\bar{x}). \quad (13)$$

$$\Pr(\bar{y}) = \Pr(\bar{y}|x)\Pr(x) + \Pr(\bar{y}|\bar{x})\Pr(\bar{x}) \geqslant \Pr(\bar{y}|x)\Pr(x) + \Pr(\bar{y}|x)(1 - \Pr(x)) = \Pr(\bar{y}|x). \quad (14)$$

Furthermore,

$$\Pr(x,y) = \Pr(x)\Pr(y|x) \geqslant \Pr(x)\Pr(y), \quad (15)$$

$$\Pr(\bar{x},\bar{y}) = \Pr(\bar{x})\Pr(\bar{y}|\bar{x}) \geqslant \Pr(\bar{x})\Pr(\bar{y}), \quad (16)$$

$$\Pr(x,\bar{y}) = \Pr(x)\Pr(\bar{y}|x) \leqslant \Pr(x)\Pr(\bar{y}), \quad (17)$$

$$\Pr(\bar{x},y) = \Pr(\bar{x})\Pr(y|\bar{x}) \leqslant \Pr(\bar{x})\Pr(y). \quad (18)$$

Therefore,

$$\log\left(\frac{\Pr(x,y)}{\Pr(x)\Pr(y)}\right) \geqslant 0, \quad \log\left(\frac{\Pr(\bar{x},\bar{y})}{\Pr(\bar{x})\Pr(\bar{y})}\right) \geqslant 0, \quad \log\left(\frac{\Pr(x,\bar{y})}{\Pr(x)\Pr(\bar{y})}\right) \leqslant 0, \quad \log\left(\frac{\Pr(\bar{x},y)}{\Pr(\bar{x})\Pr(y)}\right) \leqslant 0. \quad (19)$$

Moreover, since $\Pr(x,y) + \Pr(\bar{x},\bar{y}) + \Pr(x,\bar{y}) + \Pr(\bar{x},y) = 1$, we conclude that the higher $\Pr(x,y) + \Pr(\bar{x},\bar{y})$ is, the bigger $I(X;Y)$ is. That is, when the values of X and Y are also *true* or *false*, their MI is bigger than the value in any other cases.

Similarly, If variable X negatively influences Y, the higher $\Pr(x,\bar{y}) + \Pr(\bar{x},y)$ is, the bigger $I(X;Y)$ is. That is, when two variables X and Y obtain different values, their MI is bigger than the value in any other cases.

Further, in the following, we give a definition of qualitative MI.

Definition 3 (*Qualitative MI, QMI*). Given discrete variables X and Y, and threshold value $\beta(0.5 \leqslant \beta \leqslant 1)$,

(1) if $\delta = +$ and $\beta \leqslant \Pr(x,y) + \Pr(\bar{x},\bar{y}) \leqslant 1$, or if $\delta = -$ and $\beta \leqslant \Pr(x,\bar{y}) + \Pr(\bar{x},y) \leqslant 1$, the Qualitative MI of variables X and Y is called strong MI, denoted by $I^S(X;Y)$.
(2) if $\delta = +$ and $0.5 \leqslant \Pr(x,y) + \Pr(\bar{x},\bar{y}) < \beta$, or if $\delta = -$ and $0.5 \leqslant \Pr(x,\bar{y}) + \Pr(\bar{x},y) < \beta$, the Qualitative MI of variables X and Y is called weak MI, denoted by $I^W(X;Y)$.

From Definition 3, we have that QMI includes strong and weak MI, that is, QMI $\in \{S, W\}$.

Since the ambiguity arises from the composition of multiple non-ambiguous opposite basic signs along the parallel trails, we only need to consider the positive qualitative sign and the negative qualitative sign, the zero and ambiguous sign do not need to do here.

3.3. The formalism of QMIQPN

Based on QMI, we propose a new enhanced QPN (QMIQPN) which can distinguish between strong and weak influences, differing from the basic QPN.

Definition 4 (*Positive Influence with Strength*). Let $Q = (G, \delta)$ be a QPN, $G = (V, E)$ be a DAG, and X, Y be variables in V with $X \to Y \in E$, given threshold value $\beta(0.5 \leqslant \beta \leqslant 1)$, the influence with strength of variable X on Y is strongly positive, written $S^{++}(X,Y)$, iff

$$\delta = + \quad \text{and} \quad \beta \leqslant \Pr(x,y) + \Pr(\bar{x},\bar{y}) \leqslant 1.$$

The influence of variable X on variable Y along the arc is weakly positive, written $S^+(X,Y)$, iff

$$\delta = + \quad \text{and} \quad 0.5 \leqslant \Pr(x,y) + \Pr(\bar{x},\bar{y}) < \beta.$$

Definition 5 (*Negative Influence with Strength*). Let Q, G, X, Y, β be as in Definition 4, the influence with strength of variable X on Y is strongly negative, written $S^{-}(X,Y)$, iff

$$\delta = - \quad \text{and} \quad \beta \leqslant \Pr(\bar{x},y) + \Pr(x,\bar{y}) \leqslant 1.$$

The influence of variable X on variable Y along the arc is weakly negative, written $S^-(X,Y)$, iff

$$\delta = - \quad \text{and} \quad 0.5 \leqslant \Pr(\bar{x},y) + \Pr(x,\bar{y}) < \beta.$$

Furthermore, the definition of QMIQPN can be given below.

Definition 6 (*QMIQPN*). An enhanced QPN based on QMI (QMIQPN) $Q = (G, \delta_e)$ also comprises a DAG $G = (V, E)$ modeling variables and the probabilistic relationships between them. Instead of qualitative influences in QPN, however, the QMIQPN associates its digraph with a set $\delta_e(\delta_e \in \{++, --, +, -, 0, ?\})$ of qualitative influences with strength based on QMI.

Example 3. Now we consider the above *Radiotherapy* QPN again, the known sample data is shown in Table 2. Let $\beta = 0.75$, we have

$$0.5 < \Pr(t,s) + \Pr(\bar{t},\bar{s}) = 0.52 < \beta,$$
$$\beta < \Pr(t,r) + \Pr(\bar{t},\bar{r}) = 0.84 < 1,$$
$$0.5 < \Pr(s,\bar{l}) + \Pr(\bar{s},l) = 0.52 < \beta,$$
$$\beta < \Pr(r,l) + \Pr(\bar{r},\bar{l}) = 0.80 < 1.$$

According to Definitions 4–6, the *Radiotherapy* QMIQPN is obtained and shown by Fig. 3.

For example, from Fig. 3, we have $S^{\delta_e}(T,S) = S^{+}(T,S)$, which shows that variable T weak positively influence S. The illustrations for other influences with strengths are quite similar.

3.4. Relative properties

The properties of symmetry, transitivity and parallel composition hold for qualitative influences in a QPN. In this subsection, we address the property of symmetry of qualitative influences with strengths, followed by a discussion and enhancement of $\otimes$- and $\oplus$-operators (renamed $\otimes_e$- and $\oplus_e$-operators) provide for the properties of transitivity and parallel composition of qualitative influences with strengths, respectively. The strengths of the zero and ambiguous influences would not be considered. That is, for '0' and '?', they are the same that of in QPN.

3.4.1. Symmetry

In a QPN, the property of symmetry guarantees that, if a variable X exerts an influence on variable Y, then variable Y also exerts an influence of the same sign on variable X. However, does the symmetry property hold for the strength of an influence in a QMIQPN?

Theorem 4. *Let QMIQPN* = (G,δ_e) *and* $G = (V,E)$, *where* $X, Y \in V$ *and* $X \to Y \in E$. *Then,*

$$S^{\delta_e}(X,Y) \Longleftrightarrow S^{\delta_e}(Y,X).$$

Proof. We know, the set δ of qualitative influences in a QPN exhibits the symmetry property. That is,

$$S^{\delta}(X,Y) \Longleftrightarrow S^{\delta}(Y,X).$$

For the strengths of qualitative influences in a QMIQPN, since

$$\Pr(x,y) = \Pr(y,x), \Pr(\bar{x},\bar{y}) = \Pr(\bar{y},\bar{x}),$$
$$\Pr(\bar{x},y) = \Pr(y,\bar{x}), \Pr(x,\bar{y}) = \Pr(\bar{y},x),$$

QMI is also symmetry by Definition 3, Therefore, we conclude that

$$S^{\delta_e}(X,Y) \Longleftrightarrow S^{\delta_e}(Y,X). \qquad \square$$

Note that QMI is a special form of MI and also symmetry, which is consistent with Theorem 1. As a result, in a QMIQPN, signs ($\delta_e \in \{++,--,+,-,0,?\}$) can be propagated in both directions of an arc during inference.

Table 2
The given sample data. V denotes the variables, N denotes the sample number.

$V \setminus N$	1–5	6–10	11–15	15–20	21–25
T	0 1 1 1 1	1 1 0 1 0	1 0 0 0 0	0 1 1 1 0	1 0 0 1 1
S	0 0 0 0 1	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	1 0 0 0 0
R	0 1 1 1 1	1 1 0 0 0	1 0 0 0 1	1 1 1 1 0	1 0 1 1 1
L	0 0 0 1 0	1 1 0 0 0	1 0 0 0 1	1 1 1 1 0	1 1 1 0 1

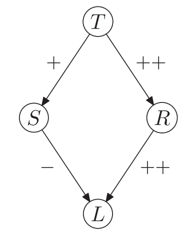

Fig. 3. The *Radiotherapy* QMIQPN.

3.4.2. Transitivity

In QMIQPN, it will be evident that the enhancement of the $\otimes$-operator (renamed $\otimes_e$-operator) need to be taken into consideration when multiplying signs. To address Sign-product of two qualitative signs, we consider the serial fragment shown in Fig. 4(a), it is a Morkov chain $X \to Y \to Z$, and includes an active trail that is composed of the variables X, Y, Z and two qualitative influences with strengths between them.

According to data-processing inequality in Theorem 2, from the serial fragment in Fig. 4(a), we also have

$$\mathrm{QMI}(X;Y) \geqslant \mathrm{QMI}(X;Z) \text{ and } \mathrm{QMI}(Y;Z) \geqslant \mathrm{QMI}(X;Z).$$

In other words,

$$\mathrm{QMI}(X;Z) \leqslant \min\{\mathrm{QMI}(X;Y), \mathrm{QMI}(Y;Z)\}.$$

Thus, for abbreviation and high efficiency upon inference, sign-product differing from the basic QPN is defined as follows.

Definition 7 (*Sign-product differing from the basic QPN*). Let QMIQPN = (G,δ_e) and $G = (V,E)$, where $X, Y, Z \in V$ and $X \to Y$, $Y \to Z$ (or $X \to Y \to Z$) are the only active trails between the variables X and Z. Then

$$S^{\delta_e}(X,Z) = S^{\delta_e^1}(X,Y) \otimes_e S^{\delta_e^2}(Y,Z) = S^{\delta_1[\mathrm{QMI}_1]}(X,Y) \otimes_e S^{\delta_2[\mathrm{QMI}_2]}(Y,Z)$$
$$= S^{\delta_1 \otimes \delta_2[\min\{\mathrm{QMI}_1,\mathrm{QMI}_2\}]}(X,Z).$$

where $\mathrm{QMI}_1, \mathrm{QMI}_2 \in \{S, W\}$, and $S > W$, $\delta_e, \delta_e^1, \delta_e^2 \in \{++,--,+,-\}$, $\delta_1, \delta_2 \in \{+,-\}$.

According to Definition 7, we have the $\otimes_e$-operator as Table 3. Table 3 now defines the $\otimes_e$-operator, which shapes the transitivity property for qualitative influences with strengths in a QMIQPN. From the table, it is readily seen that the '+', '−', '0', and '?' signs in essence combine just as in a basic QPN, the only difference is in the handling of the influence strength.

3.4.3. Composition

Similarly, to address Sign-sum of two signs in QMIQPN, we have to enhance the $\oplus$-operator (renamed $\oplus_e$-operator). We consider the parallel fragment shown in Fig. 4(b). The fragment includes two parallel active trails between the variables X and Z, one of which captures a direct influence of X on Z (the strength is denoted

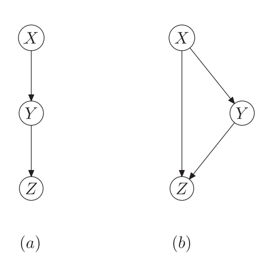

Fig. 4. Two fragments in QMIQPN. (a) Serial fragment. (b) Parallel fragment.

Table 3
The $\otimes_e$-operator.

$\otimes_e$	++	+	0	– –	–	?
++	++	+	0	– –	–	?
+	+	+	0	–	–	?
0	0	0	0	0	0	0
– –	– –	–	0	++	+	?
–	–	–	0	+	+	?
?	?	?	0	?	?	?

by QMI_1) and the other one captures an indirect influence through Y (the strength is denoted by QMI_2).

By Chain Rule for Information in Theorem 3, we have

$$\text{QMI}(X,Y;Z) = \text{QMI}(X;Z) + \text{QMI}(Y;Z|X) = \text{QMI}(Y;Z) + \text{QMI}(X;Z|Y).$$

Further,

$$\text{QMI}(X,Y;Z) \geqslant \text{QMI}_1 \text{ and } \text{QMI}(X,Y;Z) \geqslant \text{QMI}_2.$$

That is, if QMI_1 and QMI_2 are the QMI of $X \to Z$ in two parallel trail, then

$$\text{QMI}(X;Z) \geqslant \max\{\text{QMI}_1(X;Z), \text{QMI}_2(X;Z)\}.$$

Therefore, in a QMIQPN, we built the following theorem on Sign-sum of two signs differing from the basic QPN.

Theorem 5 (*Sign-sum differing from the basic QPN*).

$$S^{\delta_e}(X,Z) = S^{\delta_e^1}(X,Z) \oplus_e S^{\delta_e^2}(X,Z) = S^{\delta_1[\text{QMI}_1]}(X,Z) \oplus_e S^{\delta_2[\text{QMI}_2]}(X,Z)$$
$$= \begin{cases} S^{\delta_1 \oplus \delta_2[\max\{\text{QMI}_1,\text{QMI}_2\}]}(X,Z), & \text{if } \delta_1 = \delta_2; \\ S^{\delta_1[\text{QMI}_1]}(X,Z), & \text{if } \delta_1 \neq \delta_2 \text{ and } \text{QMI}_1 > \text{QMI}_2; \\ S^{\delta_2[\text{QMI}_2]}(X,Z), & \text{if } \delta_1 \neq \delta_2 \text{ and } \text{QMI}_1 < \text{QMI}_2; \\ S^{\delta_1 \oplus \delta_2}, & \text{if } \delta_1 \neq \delta_2 \text{ and } \text{QMI}_1 = \text{QMI}_2. \end{cases}$$

where $\text{QMI}_1, \text{QMI}_2 \in \{S, W\}$, *and* $S > W$, $\delta_e, \delta_e^1, \delta_e^2 \in \{++, --, +, -\}$, $\delta_1, \delta_2 \in \{+, -\}$.

Proof. It will be proved by means of the following Propositions 1–3. □

In order to prove Theorem 5, we will analyze the 16 different cases for the only difference in the handling of the influence strengths, as is shown in Table 4. The following several propositions show that the $\oplus_e$-operator correctly captures the sign of a combination of two parallel influences with strengths. The proofs for the other combinations of influences are quite similar. Proposition 1 addresses the case that two same qualitative influences along parallel trails are combined into a composite influence.

Proposition 1. *Let* $Q = (G, \delta_e)$ *be a QMIQPN. Let* X, Z *be variables in G and* t_1, t_2 *be parallel active trail in G from X to Z, where* $t_1 \| t_2$ *is their trail composition. Then,*

$$S^{++}(X,Z,t_1) \oplus_e S^{++}(X,Z,t_2) \Rightarrow S^{++}(X,Z,t_1\|t_2).$$

Proof. We assume that the trail t_1 consists of a single arc $X \to Z$ and the trail t_2 consists of the arcs $X \to Y$ and $Y \to Z$, as parallel fragment in Fig. 4(b). The serial influence in trail t_2 can be obtained by applying the transitivity property. Additional parallel trails between X and Z can be handled by repeated application of the composition property, and are therefore disregarded here. Given $S^{++}(X,Z,t_1)$ and $S^{++}(X,Z,t_2)$, by Definition 4, we have

$$\delta_1 = + \quad \text{and} \quad \beta \leqslant \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 1, \tag{20}$$
$$\delta_2 = + \quad \text{and} \quad \beta \leqslant \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 1. \tag{21}$$

If variables X and Z in two parallel trails take the same values, all *true* or all *false*, the probability range of variables X and Z taking the same values is $[2\beta, 2]$ other than the probability of superposition computation. Since the probability of superposition computation at most is the minimal range of formula (20) and (21), the probability range of variables X and Z taking the same values at least is the maximal range of formula (20) and (21), that is,

$$\beta \leqslant \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 1,$$

moreover, the composition of qualitative influences in a basic QPN, $\delta_1 \oplus \delta_2 = + \oplus + = +$. We therefore conclude that $S^{++}(X,Z,t_1\|t_2)$ according to Definition 4. □

From the above proposition, we have that two same qualitative influences the $\oplus_e$-operator captures the sign of their composition. Similar observations hold for the composition, such as the cases 1, 2, 3, 4, 5, 6, 7, and 8 in Table 4, of two same qualitative signs, be they weak or strong, and be they positive and negative.

Ambiguities arise in essence from combining two or more conflicting influences along parallel active trails. The next proposition provides for the combination of conflicting influences and describes the type of ambiguity that can now typically be reduced.

Proposition 2. *Let* Q, X, Z, t_1, t_2 *and* $t_1\|t_2$ *be as in the previous Proposition 1. Then,*

$$S^{++}(X,Z,t_1) \oplus_e S^{-}(X,Z,t_2) \Rightarrow S^{++}(X,Z,t_1\|t_2).$$

Proof. The proof proceeds in a similar way as the proof of Proposition 1. Given $S^{++}(X,Z,t_1)$ and $S^{-}(X,Z,t_2)$, according to Definitions 4 and 5, we know

$$\delta_1 = + \quad \text{and} \quad \beta \leqslant \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 1, \tag{22}$$
$$\delta_2 = - \quad \text{and} \quad 0.5 \leqslant \Pr(\bar{x},z) + \Pr(x,\bar{z}) < \beta. \tag{23}$$

and since

$$\Pr(x,z) + \Pr(\bar{x},\bar{z}) + \Pr(\bar{x},z) + \Pr(x,\bar{z}) = 1.$$

Furthermore, we have

$$\delta_2 = - \text{ and } 1 - \beta < \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 0.5. \tag{24}$$

If variables X and Z in two parallel trails take the same values, then the probability range of variables X and Z taking the same values is $[1, 1.5]$ other than the probability of superposition computation. Similarly, the maximal range of formula (22) and (24) is

Table 4
The 16 cases of $\oplus_e$-operator differing $\oplus$-operator.

$\oplus_e \setminus$ *cases*	1	2	3	4	5	6	7	8
$\delta_e^1 \oplus_e \delta_e^2$	$++\oplus_e++$	$++\oplus_e+$	$+\oplus_e++$	$+\oplus_e+$	$--\oplus_e--$	$--\oplus_e-$	$-\oplus_e--$	$-\oplus_e-$
$\oplus_e \setminus$ *cases*	9	10	11	12	13	14	15	16
$\delta_e^1 \oplus_e \delta_e^2$	$++\oplus_e-$	$-\oplus_e++$	$--\oplus_e+$	$+\oplus_e--$	$++\oplus_e--$	$--\oplus_e++$	$+\oplus_e-$	$-\oplus_e+$

$\beta \leqslant \Pr(x,z) + \Pr(\bar{x},\bar{z}) \leqslant 1,$

and $\delta_1 \oplus \delta_2 = + \oplus - = ?$. However, since the positive influence is known to be stronger than the conflicting negative one, we may conclude the combined influence to be positive, that is '+', thereby effectively reducing the ambiguity. We therefore have $S^{++}(X,Z,t_1\|t_2)$ according to Definition 4. □

From Proposition 2, for two different influences with different strengths, the $\oplus_e$-operator captures the sign of their composition. Similar observations hold for the composition of two different qualitative signs with different strengths, be they positive and negative, such as the cases 9, 10, 11, and 12 in Table 4.

To some extent, there are some ambiguities that do not be reduced. These cases are considered in Proposition 3.

Proposition 3. *Let* Q, X, Z, t_1, t_2 *and* $t_1\|t_2$ *be as in the previous Proposition 1. Then,*

$$S^{++}(X,Z,t_1) \oplus_e S^{--}(X,Z,t_2) \Rightarrow S^{?}(X,Z,t_1\|t_2).$$

Proof. According to the composition of qualitative influences, $+ \oplus - = ?$. Since the strongly positive influence is unknown to be stronger than the conflicting strongly negative one, we may conclude the combined influence to be ambiguous. In this case, the ambiguity could not be reduced. □

From Proposition 3, we have that for two different influences with the same strength, the $\oplus_e$-operator captures the sign of their composition. Similar observations hold for the composition of two different influences with the same strength, be they weak or strong, such as the cases 13, 14, 15, and 16 in Table 4.

Based on the above analysis, we conclude the $\otimes_e$-operator as Table 5. For '+', '−', '0', and '?' signs combine as in a basic QPN, the only difference is in the handling of the influence strengths.

3.4.4. Algebraic property of composition

Theorem 5 and Table 5 give the effective method for combining two influences with strengths. However, the order of strengths does matter for $\oplus_e$-operator while having more than two influences. That is, the $\oplus_e$-operator for combining signs of parallel influences are no longer associative, which can result in loss of information upon combining several trails having strong and weak conflicting influences. This is illustrated by the following example:

$$(++ \oplus_e +) \oplus_e - = ++ \oplus_e - = ++, ++ \oplus_e (+ \oplus_e -) = ++ \oplus_e ? = ?.$$

We stress that both combinations in this example lead to correct results, the first is just more informative than the second. Heuristics, for example, searching the influence sign with maximal strength as the first influence to combine them, can be designed to prevent unnecessary ambiguous results due to order of combination. Such heuristics can be done in $O(m)$ time, where m is the number of participating influences.

Table 5
The $\oplus_e$-operator.

$\oplus_e$	++	+	0	− −	−	?
++	++	++	++	?	++	?
+	++	+	+	− −	?	?
0	++	+	0	− −	−	?
− −	?	− −	− −	− −	− −	?
−	++	?	−	− −	−	?
?	?	?	?	?	?	?

4. Ambiguity reduction

In Section 3, we have introduced the formalism of QMIQPN and have addressed relative properties. Building upon the two enhanced operators, in this section, we first improve straightforwardly the basic *Sign-propagation Algorithm* to reduce some ambiguities, and then illustrate the application of the improved algorithm. In addition, we analyze its complexity.

4.1. The improved Sign-propagation Algorithm

In QMIQPN, the improved *Sign-propagation Algorithm* is summarized in pseudocode in Algorithm 2. In essence, the QMIQPN first can be obtained by means of (small) sample data and the known QPN, then we infer the sign of the influence with strength along all active trails, building upon the properties of symmetry, transitivity and composition of influences with strength. In the process of inference we need to search the influence sign with maximal strength as the first influence when there are more than two influences.

Algorithm 2. The improved Sign-propagation Algorithm

Input: A QPN, (small) sample data, the evidence node (the observed node) and its sign $\delta_e \in \{++, +, --, -, 0, ?\}$
Output: δ_e of each target node
Obtain QMIQPN by (small) sample data and QPN;
for each node $V_i \in V$ in QMIQPN
    $\delta_e[V_i] \leftarrow$ '0'
    PropagateSign($\emptyset, O, O, \delta_e$ *of evidence node*)
end for
Procedure PropagateSign(*trail*, *from*, *to*, *messagesign*):
$\delta_e[to] \leftarrow \delta_e[to] \oplus_e$ *messagesign*
trail $\leftarrow$ *trail* $\cup \{to\}$
for each active neighbor V_i of *to*
    linksign $\leftarrow \delta_e$ of influence between *to* and V_i
    messagesign $\leftarrow \delta_e[to] \otimes_e$ *linksign*
    Searching the influence sign with maximal strength as the first influence when more than two influences
    if $\delta_e[V_i] \notin$ *trail* and $\delta_e[V_i] \neq \delta_e[V_i] \oplus_e$ *messagesign* **then**
        PropagateSign(*trail*, *to*, V_i, *messagesign*)
    end if
end for

Example 4. Upon inference with the basic *Sign-propagation Algorithm*, we recall that entering the sign '+' for variable T results in the ambiguous sign '$- \oplus + = ?$' for variable L which is shown in Fig. 2(b). Now, we consider reducing ambiguity with the improved Sign-propagation Algorithm. First the *Radiotherapy* QMIQPN shown in Fig. 5(a) is obtained by QPN and the given data in Example 3. Since there are at most two influences to participate in composition, we do not consider the order of composition nodes in this network. Then we enter the sign '++' for variable T, reflecting a strongly positive observation for T. T propagates a message with sign '$++ \otimes_e + = +$' towards S. S updates its node-sign to '+' and sends a message with sign '$+ \otimes_e - = -$' to L. L updates its node-sign to '−' and sends no messages. T also sends a message, with sign '$++ \otimes_e ++ = ++$', to R. R updates its sign and passes a message with sign '$++ \otimes_e ++ = ++$' to L. L receives the additional sign '++'. L will now combine the signs it has received from the two parallel trails originating in T, '$- \otimes_e ++ = ++$'. The result of this combination depends on the $\otimes_e -$ operator used, thus the ambiguity in variable L can be reduced, which is shown in Fig. 5(b).

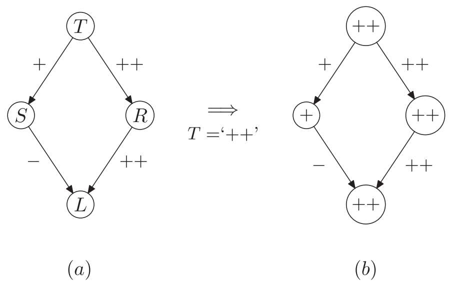

Fig. 5. The ambiguity reduction in QMIQPN. (a) *Radiotherapy* QMIQPN. (b) Give *T* = '++', the ambiguity is reduced in node *L*.

4.2. Complexity analysis

First, the symmetry property holds for influence strength in a QMIQPN, so it has lower computation complexity in obtaining QMIQPN from sample data than other EQPNs which enhancement comes at the expense of the property of symmetry of influence strength.

Secondly, as mentioned in the algebraic property of composition, the $\oplus_e$-operator for combining signs of parallel influences are no longer associative, the order of strengths does matter when combining more than two influences. We exactly desire to achieve the much informative and unique results of combing influence strengths as a measure for the given strengthen influences. Thus, we search the influence with the maximal strength as the first participating influence to combine multiple parallel influences. Searching the first composition node with maximal strength can be done in $O(m)$ time, where m is the number of participating influences. However, Yue et al. [18] adopt the non-increasing order of strengths and the composition can be done in $O(m\log m)$ time.

Thirdly, the basic *Sign-propagation Algorithm* for inference in a QPN has a worst-case runtime complexity that is polynomial in the number of nodes of the network's digraph, regardless of the digraph's topology. The main difference between our improved *Sign-propagation Algorithm* and the basic one is that, in multiply connected digraphs, the limit of two visits to each variable no longer applies. Although a variable's enhanced node-sign can change at most three times — first from '0' to '++', '−−' or '?', then '+', '−' or '?', and then only to '?', the time complexity of QMIQPN inference is still polynomial in the number of nodes of the network's digraph.

5. Experiments

In this section, we will analyze the performance of our method by experiments, including the reduction ratio of ambiguities, denoted by *RR*, and the accuracy ratio of inference results, denoted by *AR*. Specially, we definite

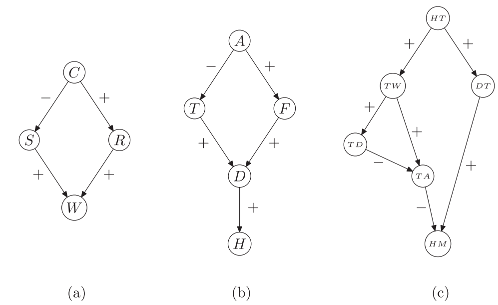

Fig. 7. Three QPNs. (a) *Lawn sprinkler* QPN. (b) *Antibiotics* QPN. (c) The simplified *Boerlage's* QPN.

$$RR = \frac{\text{the reduction number of ambiguities}}{\text{the number of ambiguities in QPN inference}}, \tag{25}$$

$$AR = \frac{\text{the number of consistent trend with BN inference}}{|\text{variables}|^2 - |\text{variables}|}. \tag{26}$$

where |variables| is the number of variables in a network.

In addition, several methods are proposed for reducing ambiguity. Because our method and EQPN based on RS as the above mentioned have the same prerequisite (QPN and data), we stress to directly compare our method with EQPN based on RS in the following analysis. First we discuss analyze on single dataset, and then on multiple datasets.

5.1. Experiment setting

All the methods have been implemented in Matlab by making use of Bayes net toolbox [31]. We analyze the performance of our method by experiments on three probabilistic networks, such as *Lawn sprinkler* network [31], *Antibiotics* network [14], and the simplified *Boerlage*'s network [32], three BNs are shown in Fig. 6. The corresponding QPNs are shown in Fig. 7.

5.2. Experimental analysis on single dataset

First we experiment on *Lawn sprinkler* network, and only consider the case of evidence value increasing. The case of evidence value decreasing is similar. For example, we regarded the evidence change from $\Pr(C=1)=0$ to $\Pr(C=1)=1$ as the increase, while the change from $\Pr(C=1)=1$ to $\Pr(C=1)=0$ as the decrease of C's *True* values in *Lawn sprinkler* BN. Likewise we could observe the trend of increase or decrease of other nodes.

According to Junction Tree Algorithm [31], the inference result of *Lawn sprinkler* BN is shown in Table 6, and that of the corresponding QPN is shown in Table 7 by using *Sign-propagation Algorithm* (see Fig. 8).

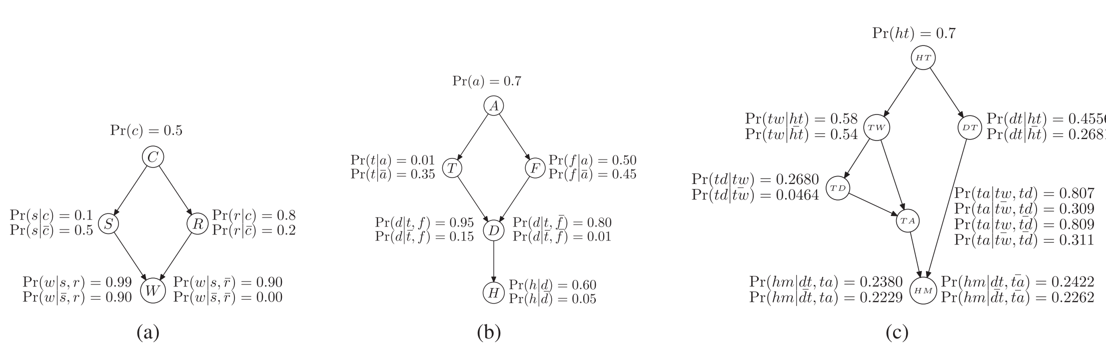

Fig. 6. Three BNs. (a) *Lawn sprinkler* BN. (b) *Antibiotics* BN. (c) The simplified *Boerlage's* BN.

Table 6
Lawn sprinkler BN inference: Junction Tree Algorithm.

Evidence	*C*	*S*	*R*	*W*
$C(0 \to 1)$	×	$0.5000 \to 0.1000$	$0.2000 \to 0.8000$	$0.5490 \to 0.7452$
$S(0 \to 1)$	$0.6429 \to 0.1667$	×	$0.5857 \to 0.3000$	$0.5271 \to 0.9270$
$R(0 \to 1)$	$0.2000 \to 0.8000$	$0.4200 \to 0.1800$	×	$0.3780 \to 0.9162$
$W(0 \to 1)$	$0.3610 \to 0.5758$	$0.0621 \to 0.4298$	$0.1187 \to 0.7079$	×

Table 7
Lawn sprinkler QPN inference: Sign propagation Algorithm.

Evidence	*C*	*S*	*R*	*W*
$C(+)$	+	–	+	?
$S(+)$	–	+	–	?
$R(+)$	+	–	+	?
$W(+)$	?	?	?	+

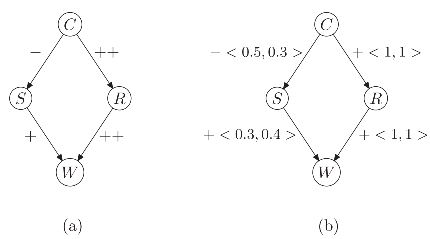

Fig. 8. *Lawn sprinkler* networks. (a) *Lawn sprinkler* QMIQPN. (b) *Lawn sprinkler* EQPN based on RS.

Further, we sample 35 data records from *Lawn sprinkler* BN. Let threshold value $\alpha = \beta = 0.75$, and then we obtain *Lawn sprinkler* QMIQPN shown in Fig. 8(a) and EQPN based on RS shown in Fig. 8(b).

According to Algorithm 2, when each evidence node increases, the inference result of QMIQPN is shown in Table 8, and that of EQPN based on RS by Algorithm in [18] is shown in Table 9.

From Tables 6–9, we can obtain *RR* and *AR* as follows:

1. *RR*. By comparing the results of the inference among *Lawnsprinkler* BN, QPN and QMIQPN or EQPN based on RS, we have that there are 6 ambiguities in Table 7, but 2 ambiguities in Table 8 and 1 ambiguity in Table 9. In a word, the *RR* is 0.6667(4/6) in QMIQPN, and that is 0.8333(5/6) in EQPN based on RS. The results show that some ambiguities can be reduced to some extent in our method and EQPN based on RS, but they cannot be reduced completely.
2. *AR*. By checking whether the trend of inference in QMIQPN or EQPN based on RS was consistent with that in BN, we verify the accuracy of results. From Table 9we have that all the results in QMIQPN inference are consistent with the trends of its BN's inference. But from Table 9 the results are not all consistent with each other, i.e. $-[0.252]$ in Table 9 indicates the trend

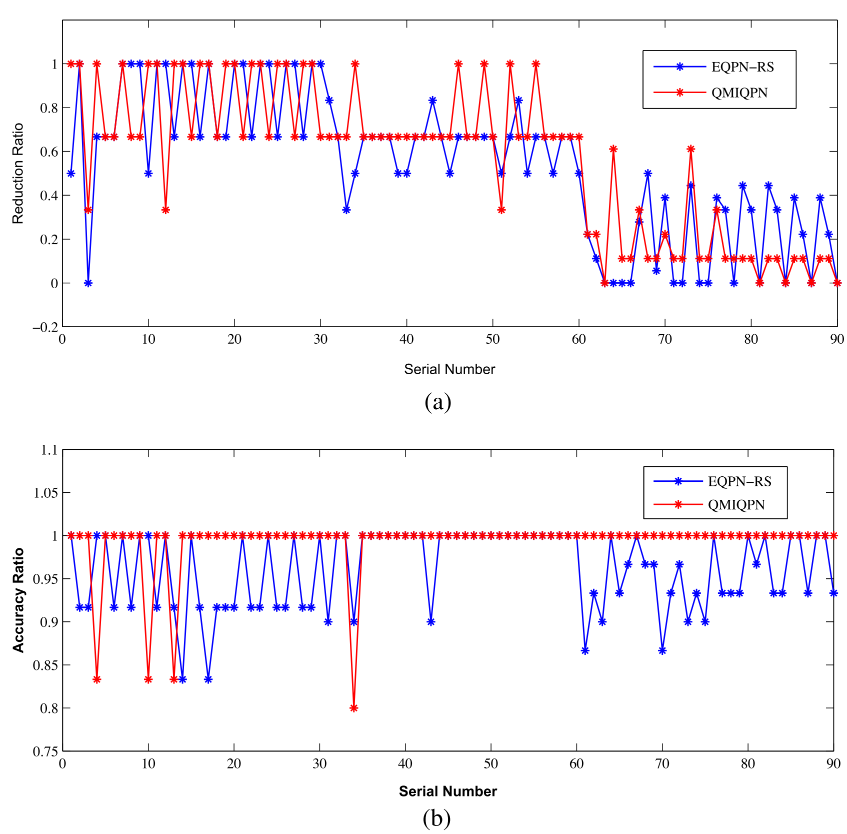

Fig. 9. The comparison of two methods. (a) The comparison of *RR*. (b) The comparison of *AR*.

Table 8
Lawn sprinkler QMIQPN inference: The Improved Sign propagation Algorithm.

Evidence	*C*	*S*	*R*	*W*
$C(++)$	++	–	++	++
$S(++)$	–	++	–	?
$R(++)$	++	–	++	++
$W(++)$	++	?	++	++

Table 9
Lawn sprinkler EQPN based on RS inference: Sign propagation Algorithm.

Evidence	*C*	*S*	*R*	*W*
$C(+[0.9])$	+[0.9]	−[0.45]	+[0.9]	+[0.8865]
$S(+[0.9])$	−[0.27]	+[0.9]	−[0.27]	?[0.27]
$R(+[0.9])$	+[0.9]	−[0.45]	+[0.9]	+[0.8865]
$W(+[0.9])$	+[0.8892]	−**[0.252]**	+[0.8892]	+[0.9]

The bold value shows the value is not consistent with the trends of its BN's inference.

decreasing, but in Table 6 the trend is increasing (that is, $0.0621 \to 0.4298$). In a word, the *AR* is 1.0000(12/12) in QMIQPN, and that is 0.9167(11/12) in EQPN based on RS. The results show that the reduction methods maybe have the wrong inference.

5.3. Experimental analysis on multiple datasets

To further analyze the performance of QMIQPN, now we discuss the experimental results on the above mentioned three datasets. Specially, in each dataset, we give different data records and different threshold values. The data records are 35, 50, 80, 100, 200, 300, 500, 1000, 3000, and 5000 in each dataset. Let threshold values $\alpha = \beta$ in each data records, they are 0.58, 0.68,0.78, respectively. Similarly, we only consider the case of evidence value increasing. Then we compare the *RR* and *AR* of our method with that of EQPN based on RS in these cases. The comparison results are shown in Table 10.

Table 10
The comparison of QMIQPN with EQPN based on RS on three datasets. *SN* denotes sample data number, α and β denote threshold value in two networks, *RR* is the reduction ratio of ambiguities, and *AR* is the accuracy ratio of inference results.

Dataset	*SN*	EQPN-RS(+[0.9])			QMIQPN(++)		
		α	*RR*	*AR*	β	*RR*	*AR*
Lawn Sprinkler	35	$\alpha_1 = 0.58$	0.5000	1.0000	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9167**	$\beta_3 = 0.78$	0.3333	1.0000
	50	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	1.0000	0.8333
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.6667	0.9167	$\beta_3 = 0.78$	0.6667	1.0000
	80	$\alpha_1 = 0.58$	1.0000	1.0000	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	100	$\alpha_1 = 0.58$	0.5000	1.0000	$\beta_1 = 0.58$	1.0000	0.8333
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.3333	1.0000
	200	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	0.8333
		$\alpha_2 = 0.68$	1.0000	0.8333	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	300	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.8333	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	0.6667	0.9167	$\beta_3 = 0.78$	0.6667	1.0000
	500	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	1000	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	3000	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	5000	$\alpha_1 = 0.58$	0.6667	0.9167	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	1.0000	0.9167	$\beta_2 = 0.68$	1.0000	1.0000
		$\alpha_3 = 0.78$	1.0000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
Antibiotics	35	$\alpha_1 = 0.58$	0.8333	0.9000	$\beta_1 = 0.58$	0.6667	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.3333	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	50	$\alpha_1 = 0.58$	0.5000	0.9000	$\beta_1 = 0.58$	1.0000	0.8000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.6667	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	80	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	0.6667	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	100	$\alpha_1 = 0.58$	0.5000	1.0000	$\beta_1 = 0.58$	0.6667	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.6667	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	200	$\alpha_1 = 0.58$	0.8333	0.9000	$\beta_1 = 0.58$	0.6667	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	300	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	1.0000	1.0000

(continued on next page)

Table 10 (*continued*)

Dataset	*SN*	EQPN-RS(+[0.9])			QMIQPN(++)		
		α	*RR*	*AR*	β	*RR*	*AR*
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.6667	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	500	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.3333	1.0000
	1000	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	0.8333	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	3000	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	1.0000	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
	5000	$\alpha_1 = 0.58$	0.6667	1.0000	$\beta_1 = 0.58$	0.6667	1.0000
		$\alpha_2 = 0.68$	0.6667	1.0000	$\beta_2 = 0.68$	0.6667	1.0000
		$\alpha_3 = 0.78$	0.5000	1.0000	$\beta_3 = 0.78$	0.6667	1.0000
Boerlage's simplified network	35	$\alpha_1 = 0.58$	0.2222	0.8667	$\beta_1 = 0.58$	0.2222	1.0000
		$\alpha_2 = 0.68$	0.1111	0.9333	$\beta_2 = 0.68$	0.2222	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9000**	$\beta_3 = 0.78$	0.0000	1.0000
	50	$\alpha_1 = 0.58$	0.0000	1.0000	$\beta_1 = 0.58$	0.6111	1.0000
		$\alpha_2 = 0.68$	**0.0000**	**0.9333**	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9667**	$\beta_3 = 0.78$	0.1111	1.0000
	80	$\alpha_1 = 0.58$	0.2778	1.0000	$\beta_1 = 0.58$	0.3333	1.0000
		$\alpha_2 = 0.68$	0.5000	0.9667	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	0.0556	0.9667	$\beta_3 = 0.78$	0.1111	1.0000
	100	$\alpha_1 = 0.58$	0.3889	0.8667	$\beta_1 = 0.58$	0.2222	1.0000
		$\alpha_2 = 0.68$	**0.0000**	**0.9333**	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9667**	$\beta_3 = 0.78$	0.1111	1.0000
	200	$\alpha_1 = 0.58$	0.4444	0.9000	$\beta_1 = 0.58$	0.6111	1.0000
		$\alpha_2 = 0.68$	**0.0000**	**0.9333**	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9000**	$\beta_3 = 0.78$	0.1111	1.0000
	300	$\alpha_1 = 0.58$	0.3889	1.0000	$\beta_1 = 0.58$	0.3333	1.0000
		$\alpha_2 = 0.68$	0.3333	0.9333	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9333**	$\beta_3 = 0.78$	0.1111	1.0000
	500	$\alpha_1 = 0.58$	0.4444	0.9333	$\beta_1 = 0.58$	0.1111	1.0000
		$\alpha_2 = 0.68$	0.3333	1.0000	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9667**	$\beta_3 = 0.78$	0.0000	1.0000
	1000	$\alpha_1 = 0.58$	0.4444	1.0000	$\beta_1 = 0.58$	0.1111	1.0000
		$\alpha_2 = 0.68$	0.3333	0.9333	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9333**	$\beta_3 = 0.78$	0.0000	1.0000
	3000	$\alpha_1 = 0.58$	0.3889	1.0000	$\beta_1 = 0.58$	0.1111	1.0000
		$\alpha_2 = 0.68$	0.2222	1.0000	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9333**	$\beta_3 = 0.78$	0.0000	1.0000
	5000	$\alpha_1 = 0.58$	0.3889	1.0000	$\beta_1 = 0.58$	0.1111	1.0000
		$\alpha_2 = 0.68$	0.2222	1.0000	$\beta_2 = 0.68$	0.1111	1.0000
		$\alpha_3 = 0.78$	**0.0000**	**0.9333**	$\beta_3 = 0.78$	0.0000	1.0000

From Table 10, we can obtain the comparison of *RR* shown in Fig. 9(b) and the comparison of *AR* Fig. 9(b) in two methods under 90 cases.

Fig. 9(a) and Fig. 9(b) from Table 10, we can obtain the following conclusions:

1. From Table 10 and Fig. 9(a), we have that *RR* depends on the threshold value α or β. Let $\alpha = \beta$, for 30 cases in *Lawn Sprinkler* network, there are 22 cases that *RR* in QMIQPN is higher than or equal to that in EQPN based on RS, that is, the percentage of QMIQPN performs better than EQPN based on the RS about *RR* is 73%(22/30); for 30 cases in *Antibiotics* network, the percentage is 87%(26/30); for 30 cases in *Boerlage's simplified* network, the percentage is 60%(18/30). Thus for every dataset, QMIQPN has higher *RR* than EQPN based on RS. In addition, the percentage is lower in *Boerlage's simplified* network than that in *Lawn sprinkler* network and *Antibiotics* network. The reasons are as follows:

   (a) On the one hand, for abbreviation and high efficiency upon inference, we give Definition 7 as $QMI(X;Z) = \min\{QMI(X;Y), QMI(Y;Z)\}$, in fact, $QMI(X;Z) \leqslant \min\{QMI(X;Y), QMI(Y;Z)\}$.

   (b) On the other hand, Because in *Lawnsprinkler* network and *Antibiotics* network, there is only one $\oplus_e$-operator, but in *Boerlage's simplified* network, there are two serial $\oplus_e$-operators.

   Thus, our method is more suitable for dealing with the ambiguity which has arisen by one $\oplus_e$-operator. For more than two serial $\oplus_e$-operators, we can find the actual ambiguity that resides with one $\oplus_e$-operator and then deal with it.

2. From Table 10 and Fig. 9(b), in all 90 cases, we have that 86 cases that *AR* in QMIQPN is higher than or equal to that in EQPN based on RS. That is, the percentage of QMIQPN performs better than EQPN based on the RS about *AR* is 96%(86/90). Thus in most cases, QMIQPN has much higher *AR* than that EQPN based on RS in three networks. But QMIQPN has also a few error inference, the main reason is, for abbreviation and high efficiency upon inference, $QMI(X;Z) = \min\{QMI(X;Y), QMI(Y;Z)\}$ in Definition 7 is defined, in fact, $QMI(X;Z) \leqslant \min\{QMI(X;Y), QMI(Y;Z)\}$.

3. More important, in Table 10, there are 13 cases of $RR = 0$ and $AR \neq 0$ marked in bold in EQPN based on RS. That is, EQPN based on RS can also generate error inferences where the actual unambiguity resides, but QMIQPN cannot generate them.

6. Conclusion

To measure the qualitative influence and reduce the ambiguity upon inference in QPN, we introduce QMI as indicators of influence strengths to avoid some disadvantages in other reduction methods. The main contributions of this paper can be summarized as follows:

- We give a strict definition of QMI.
- We present a QMIQPN based on definition of QMI, and analyze its relative properties. Specially, the symmetry holds for the influence strength.
- We improve the basic *Sign-propagation Algorithm* to reduce ambiguity to some extent, the complexity of eliciting order of composition nodes is $(O(m))$ and computation complexity is polynomial in the node number in the network, where m is the node number of participating composition.
- By experiments on single and multiple databases respectively, we discuss and analyze the performance of QMIQPN.

In brief, theoretic analysis and experimental results show that QMIQPN is qualitative and efficient, yet allows for reducing some ambiguities correctly in most cases.

The method proposed in this paper also raises some other interesting research issues. Based on QMIQPN, we can further study the adaptive threshold value for ambiguity reduction, and improve the reduction rate of ambiguity not only in simple topology networks but also in complex ones or having more than two serial $\oplus_e$-operators. As well, the integration of multiple QPNs by reducing ambiguities as many as possible need to be further exploited. These are exactly our future work.

Acknowledgments

We would like to thank the anonymous reviewers for their valuable comments and suggestions. This work was supported in part by the National Natural Science Foundation of China (Nos. 60873100, 61322211), Program for New Century Excellent (No. 20120301) and the Natural Science Foundation of Shanxi Province, China (Nos. 2013011016-4, 2014011022-2).

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.knosys.2014.07.014.

References

[1] M.P. Wellman, Fundamental concepts of qualitative probabilistic networks, Artif. Intell. 44 (1990) 257–303.
[2] J. Pearl, Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference, Morgan Kaufman Publishers, San Mateo,California, 1988.
[3] A. Garcia-Crespo, B. Ruiz-Mezcua, J. Lopez-Cuadrado, I. Gonzalez-Carrasco, Semantic model for knowledge representation in e-business, Knowl.-Based Syst. 24 (2) (2011) 282–296.
[4] H. Yao, L. Etzkorn, Automated conversion between different knowledge representation formats, Knowl.-Based Syst. 19 (2006) 404–412.
[5] M. Druzdzel, M. Henrion, Efficient reasoning in qualitative probabilistic networks, in: Proceedings of the Eleventh Annual Conference on Artificial Intelligence, AAAI Press, Menlo Park, CA, 1993, pp. 548–553.
[6] G.F. Cooper, The computational complexity of probabilistic inference using Bayesian belief networks, Artif. Intell. 42 (2) (1990) 393–405.
[7] M. Wellman, Graphical inference in qualitative probabilistic networks, Networks 20 (1990) 687–701.
[8] S. Parsons, Refining reasoning in qualitative probabilistic networks, in: Proceedings of the Eleventh Conference on Uncertainty in Artificial Intelligence, Morgan Kaufman, San Francisco, California, 1995.
[9] S. Renooij, Qualitative Approaches to Quantifying Probabilistic Networks, Ph.D. thesis, Utrecht University, 2001.
[10] D. Duckham, J. Lingham, K. Mason, M. Worboys, Qualitative reasoning about consistency in geographic information, Inform. Sci. 176 (6) (2006) 601–627.
[11] P.J. Lucas, Bayesian network modelling through qualitative patterns, Artif. Intell. 163 (2005) 233–263.
[12] A. Osseiran, Qualitative Bayesian networks, Inform. Sci. 131 (2001) 87–106.
[13] A. Feelders, A new parameter learning method for Bayesian networks with qualitative influences, in: Proceedings of Uncertainty in Artificial Intelligence, AUAI Press, Corvallis (Oregon), 2007, pp. 117–124.
[14] S. Renooij, L.V. der Gaag, Enhanced qualitative probabilistic networks for resolving trade-offs, Artif. Intell. 172 (2008) 1470–1494.
[15] K. Yue, W. Liu, X. Wang, A. Zhou, J. Li, Discovering semantic associations among web services based on the qualitative probabilistic network, Exp. Syst. Appl. 36 (5) (2009) 9082–9094.
[16] W. Liu, K. Yue, S. Liu, Y. Sun, Qualitative-probabilistic-network-based modeling of temporal causalities and its application to feedback loop identification, Inform. Sci. 178 (7) (2008) 1803–1824.
[17] F. van Kouwen, S. Renooij, P. Schot, Inference in qualitative probabilistic networks revisited, Int. J. Approx. Reason. 50 (5) (2009) 708–720.
[18] K. Yue, Y. Yao, J. Li, W.Y. Liu, Qualitative probabilistic networks with reduced ambiguities, Appl. Intell. 33 (2) (2010) 159–178.
[19] K. Yue, W.Y. Liu, Quantifying influences in the qualitative probabilistic network with interval probability parameters, Appl. Soft Comput. 11 (1) (2011) 1135–1143.
[20] J. Bolt, L.V. der Gaag, S. Renooij, Introducing situational signs in qualitative probabilistic networks, Int. J. Approx. Reason. 38 (3) (2005) 333–354.
[21] C. Liu, M.P. Wellman, Bounding probabilistic relationships in Bayesian networks using qualitative influences: methods and applications, Int. J. Approx. Reason. 36 (1) (2004) 31–73.
[22] S. Renooij, S. Parsons, P. Pardieck, Using kappas as indicators of strength in qualitative probabilistic networks, in: Proceedings of the Seventh European Conference on Symbolic and Qualitative Approaches to Reasoning with Uncertainty, Springer, Berlin, 2003, pp. 87–99.
[23] C. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423. 623–656.
[24] C. Shannon, W. Weaver, The Mathematical Theory of Communication, University of Illinois Press, 1949.
[25] J. Cheng, R. Greiner, J. Kelly, D. Bell, W. Liu, Learning Bayesian networks from data: an information-theory based approach, Artif. Intell. 137 (2002) 43–90.
[26] H. Peng, F. Long, C. Ding, Feature selection based on mutual information criteria of max-dependency, max-relevance, and min-redundancy, IEEE Trans. Patt. Anal. Mach. Intell. 27 (8) (2005) 1226–1238.
[27] S. Deng, Z. He, X. Xu, G-anmi: a mutual information based genetic clustering algorithm for categorical data, Knowl.-Based Syst. 23 (2010) 144–149.
[28] M. Druzdzel, M. Henrion, Intercausal reasoning with uninstantiated ancestor nodes, in: Proceedings of the Ninth Conference on Uncertainty in Artificial Intelligence, Morgan Kaufman Publishers, San Francisco, California, 1993, pp. 317–325.
[29] J. Kwisthouta, G. Tel, Complexity results for enhanced qualitative probabilistic networks, Int. J. Approx. Reason. 48 (2008) 879–888.
[30] T.M. Cover, J.A. Thomas, Elements of Information Theory, second ed., A John Wiley and Sons, Inc., 2006.
[31] K.P. Murphy, The Bayes net toolbox for matlab, Comput. Sci. Statist. 33 (2001) 1–20 (software is available on-line <http://bnt.sourceforge.net/>).
[32] B. Boeralge, Link Strength in Bayesian Networks, Master's thesis, The University of British Columbia, 1995.