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RESEARCH ARTICLE

Comparative study of decision performance of decision tables induced by attribute reductions

WEI WEI†‡, JIYE LIANG†‡*, YUHUA QIAN†‡, FENG WANG†‡ and CHUANGYIN DANG§

†*Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, Shanxi University, Taiyuan, 030006, China*;
‡*School of Computer and Information Technology, Shanxi University, Taiyuan, 030006, Shanxi, China*;
§*Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong*

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An attribute reduction approach given decides the decision performance of a reduced decision table, which can give a guidance for selecting one rule-extraction method in practical applications. The objective of this study is to compare the decision performance of positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction. In this paper, the relationships in-between positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction are first investigated. Then, by means of three evaluation indices (certainty measure, consistency measure, and support measure), we systemically analyze these change mechanism of decision performance of a decision table induced by each of these three types of reduction approaches. Finally, by numerical experiments, these change mechanism of a decision table's decision performance are verified for the above three attribute reductions.

Keywords: Rough set theory; Attribute reduction; Decision performance evaluation; Information entropy

1. Introduction

Rough set theory was proposed by Pawlak in 1982. Recently, it has become a popular mathematical framework for pattern recognition, image processing, feature selection, neuro computing, conflict analysis, decision support, data mining and knowledge discovery process from large data sets (Bazan *et al.* 2003, Pal *et al.* 2001, Pawlak 1991, 1998, 2005, Pawlak and Skowron 2007).

In recent years, more attention has been paid to attribute reduction in information systems and decision tables. Many types of attribute reduction techniques have been proposed in the last twenty years (Beynon 2001, Düntsch and Gediaga 1998, Hu and Cercone 1995, Li *et al.* 2004, Liang and Xu 2002, Mi *et al.* 2003, Nguyen and Slezak 1999, Pawlak 1991, 1998, Quafatou 2000, Slezak 1996, 1998, Wang 2003, Wang *et al.* 2005, Wu *et al.* 2005, Yao 2008, 2003, Yao *et al.* 1999, Zhu and Wang 2003, Ziarko 1993). For our development, we briefly recall some of these techniques. Skowron (Quafatou and Rauszer 1992) proposed an attribute reduction

*Corresponding author. Email: ljy@sxu.edu.cn.

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algorithm using a discernibility matrix, which can find all reducts. However, it only works in small data sets because the algorithm is very time-consuming.

It is well known that a special heuristic function is usually used to acquire one of all reducts, which may be a tolerable strategy when only one reduct is needed. To date, several heuristic reduction approaches have been presented. Hu (Hu and Cercone 1995) used an attribute dependence to establish an heuristic algorithm for attribute reduction, which can remain all certain rules. Wang (Wang 2003, Wang *et al.* 2005) applied Shannon's information entropy for estimating the significance of an attribute. The reduction algorithm determined by this measure can also obtain one reduct, in which the certainty measure of every decision-rule derived from the decision table isn't changed. Liang (Liang *et al.* 2002, 2004, 2005, 2006, Liang and Xu 2002, Liang and Qian 2008) proposed a new uncertainty measure to information systems, and it can be employed to compute an attribute reduct of a decision table. The β-reduct proposed by Ziarko (Ziarko 1993) provides a kind of attribute-reduction methods in the variable precision rough set model. The α-reduct and α-relative reduct that allow the occurrence of additional inconsistency were proposed in (Nguyen and Slezak 1999) for information systems and decision tables, respectively. An attribute-reduction method that preserves the class membership distribution of all objects in information systems was proposed by Slezak (Slezak 1996, 1998). Five kinds of attribute reducts and their relationships inconsistent systems were investigated by Kryszkiewicz (Kryszkiewicz 2001), Li (Li *et al.* 2004) and Mi Mi *et al.* (2003), respectively. By eliminating some rigorous conditions required by the distribution reduct, a maximum distribution reduct was introduced by Mi (Mi *et al.* 2003). Unlike the possible reduct in (Mi *et al.* 2003), the maximum distribution reduct can derive decision rules that are compatible with the original system. In these reduction approaches, the reduction based on the positive-region, the reduction method based on Shannon's entropy and that based on Liang's entropy are three representative reduction approaches. They are mainly focused on in the present study.

A set of decision rules can be generated from a decision table by adopting any kind of reduction method (Huynh and Nakamori 2005, Hu and Cercone 1995, Quafatou and Rauszer 1992, Quafatou 1995, Wang 2003, Wang *et al.* 2005). In (Düntsch and Gediaga 1998), based on information entropy, Düntsch suggested some uncertainty measures of a decision rule and proposed three criteria for model selection. Moreover, several other measures such as certainty measure and support measure are often used to evaluate a decision rule (Greco *et al.* 2004, Liang *et al.* 2006). However, all of these measures are only defined for a single decision rule and are not suitable for measuring the decision performance of a rule set. There are two more kinds of measures in the literature (Pawlak 1998), which are approximation accuracy for decision classification and consistency degree for a decision table. Although these two measures, in some sense, could be regarded as measures for evaluating the decision performance of all decision rules generated from a decision table, they have some limitations. For instance, the certainty measure and consistency of a decision table be well characterized by the approximation accuracy and consistency degree for a degree for a decision table when their values reaches zero. To overcome the shortcomings of the existing measures, in the literature (Qian *et al.* 2008a,b,c), three new measures are proposed for this objective, which are certainty measure (α), consistency measure (β), and support measure (γ). These three measures can be use to evaluate the entire decision performance of a given complete and incomplete decision table.

The decision table induced by an attribute reduction still retains the indispensable attributes of the original one through eliminating the redundant attributes.

However, the decision performance of the decision table may be changed after each of attribute reductions. In this paper, we compare the changes of decision performance after attribute reductions based on positive region, on Shannon's entropy, and on Liang's entropy.

The rest of this paper is organized as follows. Some preliminary concepts are briefly recalled in Section 2. In Section 3, the relationships among positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction are investigated. In Section 4, through reviewing three existing measures for decision evaluation, the change mechanism of each of these three criteria is discovered in a decision table. In Section 5, the change of decision performance of a decision table induced by each of three existing types of reduction approaches is systemically analyzed. In Section 6, we also have employed a real data set from UCI database for experimental analysis. Experimental results show the correctness of the change mechanism obtained in this paper. Section 7 concludes this paper.

2. Preliminaries

In this section, we review some basic concepts such as indiscernibility relation, partition, decision tables, decision rules, certainty degree and support degree of a rule and the definition of reduction.

An information system (sometimes called a data table, an attribute-value system, a knowledge representation system, etc.), as a basic concept in rough set theory, provides a convenient framework for the representation of objects in terms of their attribute values.

Let $S = (U, A)$ be an information system, where U is a non-empty and finite set of objects, called a universe, and A is a non-empty and finite set of attributes. For each $a \in A$, a mapping $a : U \rightarrow V_a$ is determined by an information system, where V_a is the set of all possible values of a.

Each non-empty subset $B \subseteq A$ determines an indiscernibility relation in the following way, $R_B = \{(x, y) \in U \times U \mid a(x) = a(y), \forall a \in B\}$, where $a(x)$ and $a(y)$ respect the value of object x and y on attribute a respectively. The relation R_B partitions U into some equivalence classes given by $U/R_B = \{[x]_B \mid x \in U\}$, where $[x]_B$ denotes the equivalence class determined by x with respect to B, i.e., $[x]_B = \{y \in U \mid (x, y) \in R_B\}$. The partition U/R_B is further denoted as U/B. Furthermore, for any $Y \subseteq U$, one defines that $(\overline{B}(Y), \underline{B}(Y))$ is the rough set of Y with respect to B, where the lower approximation $\underline{B}(Y)$ and the upper approximation $\overline{B}(Y)$ of Y are described by

$$\underline{B}(Y) = \{x | [x]_B \subseteq Y\}, \text{ and}$$
$$\overline{B}(Y) = \{x | [x]_B \cap Y \neq \emptyset\}.$$

We define a partial relation $\preceq$ on the family $\{U/B \mid B \subseteq A\}$ as follows: $U/P \preceq U/Q$ (or $U/Q \succeq U/P$) if and only if, for every $P_i \in U/P$, there exists $Q_j \in U/Q$ such that $P_i \subseteq Q_j$, where $U/P = \{P_1, P_2, \cdots, P_m\}$ and $U/Q = \{Q_1, Q_2, \cdots, Q_n\}$ are partitions induced by P, $Q \subseteq A$, respectively. In this case, we say that Q is coarser than P, or P is finer than Q. If $U/P \preceq U/Q$ and $U/P \neq U/Q$, we say Q is strictly coarser than P (or P is strictly finer than Q), denoted by $U/P \prec U/Q$ (or $U/Q \succ U/P$).

Let $S = (U, C \cup D)$ with $C \cap D = \emptyset$ be an information system, where an element of C is called a condition attribute, C is called a condition attribute set, an element of D is called a decision attribute, and D is called a decision attribute set, then

S is defined as a decision table. For example, a decision table about diagnosing rheum is given by Table 1, in which $U = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ is the universe, $C = \{c_1, c_2, c_3, c_4\} = \{Headache, Muscle\ pain, Animal\ heat, Cough\}$ is the condition attribute set, and $D = \{d\} = \{Rheum\}$ is the decision attribute set.

If $U/C \preceq U/D$, then $S = (U, C \cup D)$ is said to be consistent, otherwise it is said to be inconsistent. Certain decision rules can be extracted from a consistent decision table, and both uncertain decision rules and certain decision rules can be extracted from an inconsistent decision table. Furthermore, we call the set of these condition classes which are the hypotheses of certain decision rules the consistent part of a decision table, and call the set of all other condition classes the inconsistent part of the decision table. It will be indicated by a example.

Example 2.1 From Table 1, we can find that it is a inconsistent table. Moreover, it is obvious that the set $\{e_{10}\}$ is the consistent part of Table 1, the set $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$ is the inconsistent part of Table 1.

Let $S = (U, C \cup D)$ be a decision table, $X_i \in U/C$ and $Y_j \in U/D$. By $des(X_i)$ and $des(Y_j)$, we denote the descriptions of the equivalence classes X_i and Y_j in the decision table S. A decision rule is formally defined as (Liang *et al.* 2006, Pawlak 1991):

$$Z_{ij} : des(X_i) \rightarrow des(Y_j). \tag{1}$$

The certainty degree μ and support degree s of a decision rule Z_{ij} are defined as follows (Liang *et al.* 2006, Pawlak 1991):

$$\mu(Z_{ij}) = |X_i \cap Y_j|/|X_i| \text{ and } s(Z_{ij}) = |X_i \cap Y_j|/|U|, \tag{2}$$

where $|\cdot|$ is the cardinality of a set. It is clear that the value of each of $\mu(Z_{ij})$ and $s(Z_{ij})$ of a decision rule Z_{ij} falls into the interval $[\frac{1}{|U|}, 1]$. In subsequent discussions, we denote the cardinality of the set $X_i \cap Y_j$ by $|Z_{ij}|$, which is called the support number of the rule Z_{ij}.

Let $S = (U, C \cup D)$ be a decision table, the relative positive region D with respect to C is defined as (Pawlak 1991)

$$POS_C(D) = \bigcup_{i=1}^{n} \underline{C}Y_i, \tag{3}$$

where, $Y_i \in U/D$, $\underline{C}Y_i$ indicates the lower approximation of Y_i with respect to C. Using this denotation, one can give the definition of a positive-region reduct as follows.

Definition 2.1: (Hu and Cercone 1995) Let $S = (U, C \cup D)$ be a decision table and $B \subseteq C$. We call B a positive-region reduct of D with respect to C if B satisfies the following conditions:

$$(1) \quad POS_C(D) = POS_B(D);\ and$$
$$(2) \quad for\ \forall a \in B, \quad POS_B(D) \neq POS_{B-\{a\}}(D).$$

In (Wang 2003, Wang *et al.* 2005), Shannon's condition entropy of condition attribute set C with respect to decision attribute set D in a decision table $S =$

$(U, C \cup D)$ is defined as

$$H(D|C) = -\sum_{i=1}^{m} \frac{|X_i|}{|U|} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|X_i|} \log_2 \frac{|X_i \cap Y_j|}{|X_i|}, \tag{4}$$

where $X_i \in U/C$ and $Y_j \in U/D$.

Definition 2.2: (Wang 2003, Wang *et al.* 2005) Let $S = (U, C \cup D)$ be a decision table, $B \subseteq C$. We call B a Shannon entropy reduct of D with respect to C if B satisfies the following conditions:

$$(1)\quad H(D|C) = H(D|B); and$$
$$(2)\quad for\ \forall a \in B, \quad H(D|B) \neq H(D|B - \{a\}).$$

In Literature (Liang *et al.* 2002, 2004, 2006, Liang and Qian 2008), Liang's entropy of condition attribute set C with respect to decision attribute set D in a decision table $S = (U, C \cup D)$is depicted by as

$$E(D|C) = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|X_i|} \frac{|Y_j^c - X_i^c|}{|X_i|}, \tag{5}$$

where, Y_j^c and X_i^c are the complements of Y_j and X_i, respectively.

In terms of this description, one can give the definition of a Liang's entropy reduct as follows.

Definition 2.3: Let $S = (U, C \cup D)$ be a decision table, $B \subseteq C$. We call B a Liang's entropy reduct of D with respect to C if B satisfies the following conditions:

$$(1)\quad E(D|C) = E(D|B);\ and$$
$$(2)\quad for\ \forall a \in B, \quad E(D|B) \neq E(D|B - \{a\}).$$

Positive-region, Shannon's entropy, and Liang's entropy are usually applied for the attribute reduction of a decision table.

3. Relationships among three kinds of reductions

In this section, we will analyze the relationships among positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction.

The rough monotonicity of Shannon's information entropy have been proved (Wang 2003, Wang *et al.* 2005), which is shown as follows.

Theorem 3.1: *(Wang 2003, Wang et al. 2005) Let* $S = (U, C \cup D)$ *and* $S' = (U, B \cup D)$ *be two decision tables,* $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ *and* $U/D = \{Y_1, Y_2, \cdots, Y_n\}$, *then*

$$H(D|B) \geq H(D|C),$$

especially, if and only if $\frac{|X_u \cap Y_j|}{|X_u|} = \frac{|X_v \cap Y_j|}{|X_v|}$ *for* $j \leq n$, *i.e.,* $\mu(Z_{uj}) = \mu(Z_{vj})$ *for*

$j \leq n$.

$$H(D|B) = H(D|C).$$

From Theorem 3.1, we can see that Shannon's entropy of a decision table will be not more than the one of the table with the coarser partition.

For convenience, suppose that $RED_D^P(C)$ is the set of all positive-region reduct, $B(P) \in RED_D^P(C)$ a positive-region reduct, $RED_D^S(C)$ the set of all Shannon's entropy reduct, $B(S) \in RED_D^S(C)$ a Shannon's entropy reduct, $RED_D^L(C)$ the set of all Liang's entropy reduct and $B(L) \in RED_D^L(C)$ a Liang's entropy reduct. In the following, we establish the relationship among positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction with four theorems and four corollaries.

Theorem 3.2: *(Wang 2003, Wang et al. 2005) Let $S = (U, C \cup D)$ be a decision table. If an attribute set $B(S)$ be a Shannon's entropy reduct, then there exists a positive-region reduct $B(P)$ such that $B(P) \subseteq B(S)$.*

Proof: Let $B(S)$ be a Shannon's entropy reduct, thus $H(D|C) = H(D|B(S))$. From Theorem 3.1, it follows that $\frac{|X_u \cap Y_j|}{|X_u|} = \frac{|X_v \cap Y_j|}{|X_v|}$, $j \leq n$, then, $POS_C(D) = POS_{B(S)}(D)$. Furthermore, it is certain to exist a set $B(P) \subseteq B(S)$, which satisfies $\forall a \in B(P)$, $POS_{B(P)}(D) \neq POS_{B(P)-\{a\}}(D)$. Therefore, there exists $B(P)$ is a positive-region reduct. □

By Theorem 3.2, it follows that for a decision table, there exists a subset of its Shannon's entropy reducts which is a positive-region reduct.

Corollary 3.3: *(Wang 2003, Wang et al. 2005) Let $S = (U, C \cup D)$ be a decision table, $RED_D^S(C)$ a set of all Shannon's entropy reducts, and $RED_D^P(C)$ a set of all positive-region reducts, then $min\{|B(P)| : B(P) \in RED_D^P(C)\} \leq min\{|B(S)| : B(S) \in RED_D^S(C)\}$.*

Corollary 3.3 shows that for a decision table, the cardinality of the minimum Shannon's entropy reduct is not less than the cardinality of the minimum positive-region entropy reduct.

Theorem 3.4: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$, then*

$$E(D|B) \geq E(D|C),$$

especially, if and only if $\mu(Z_{uw}) = \mu(Z_{vw}) = 1$ for $w \leq n$ and $\mu(Z_{uj}) = \mu(Z_{vj}) = 0$ for $j \leq n$ and $j \neq w$, then

$$E(D|B) = E(D|C).$$

Proof: For the existing condition, one has that

$$\begin{aligned}
E_{\Delta} &= E(D|B) - E(D|C) \\
&= \sum_{j=1}^{n} \frac{|X_u \cap Y_j| + |X_v \cap Y_j|}{|U|} \frac{|X_u - Y_j| + |X_v - Y_j|}{|U|} \\
&\quad - \sum_{j=1}^{n} \frac{|X_u \cap Y_j|}{|U|} \frac{|X_u - Y_j|}{|U|} - \sum_{j=1}^{n} \frac{|X_v \cap Y_j|}{|U|} \frac{|X_v - Y_j|}{|U|} \\
&= \sum_{j=1}^{n} \frac{|X_u \cap Y_j|}{|U|} \frac{|X_v| - |X_v \cap Y_j|}{|U|} + \sum_{j=1}^{n} \frac{|X_v \cap Y_j|}{|U|} \frac{|X_u| - |X_u \cap Y_j|}{|U|} \\
&= \sum_{j=1}^{n} \frac{|X_u||X_v|(\mu(Z_{uj}) + \mu(Z_{vj}) - 2\mu(Z_{uj})\mu(Z_{vj}))}{|U|^2}.
\end{aligned}$$

Let $f_j = \mu(Z_{uj}) + \mu(Z_{vj}) - 2\mu(Z_{uj})\mu(Z_{vj})$. It is clear that $0 \leq \mu(Z_{uj}) \leq 1$ and $0 \leq \mu(Z_{vj}) \leq 1$. The sign of f_j will be investigated as follows.

If $\mu(Z_{uj}) = 0$ and $0 < \mu(Z_{vj}) \leq 1$ (or $0 < \mu(Z_{uj}) \leq 1$ and $\mu(Z_{vj}) = 0$), then $f_j > 0$.

If $\mu(Z_{uj}) = 0$ and $\mu(Z_{vj}) = 0$, then $f_j = 0$.

If $\mu(Z_{uj}) = 1$ and $0 \leq \mu(Z_{vj}) < 1$ (or $0 \leq \mu(Z_{uj}) < 1$ and $\mu(Z_{vj}) = 1$), then $f_j > 0$.

If $\mu(Z_{uj}) = 1$ and $\mu(Z_{vj}) = 1$, then $f_j = 0$.

If $0 < \mu(Z_{uj}) < 1$ and $0 < \mu(Z_{vj}) < 1$, then $f_j > 0$.

From the above several cases, we have that $f_j \geq 0$. Then $E_{\Delta} = \sum_{j=1}^{n} \frac{|X_u||X_v|f_j}{|U|^2} \geq 0$. Furthermore, one has that $E_{\Delta} = 0$ iff $f_j = 0$. In other words, $E(D|B) = E(D|C)$ holds, if and only if $\mu(Z_{uw}) = \mu(Z_{vw}) = 1$ for $w \leq n$ and $\mu(Z_{uj}) = \mu(Z_{vj}) = 0$ for $j \leq n$ and $j \neq w$. □

Theorem 3.4 indicates that Liang's entropy of a decision table will be not more than the one of the table with the coarser condition attributes set.

Theorem 3.5: *Let $S = (U, C \cup D)$ be a decision table. If an attribute set $B(L)$ is a Liang's entropy reduct, then there exists a Shannon's entropy reduct $B(S)$ such that $B(S) \subseteq B(L)$.*

Proof: Since $B(S)$ is a Shannon's entropy reduct, we have that $E(D|C) = E(D|B(L))$. And from Theorem 3.4, it follows that there exists $w \leq n$ such that $\mu(Z_{uw}) = \mu(Z_{vw}) = 1$ and $\mu(Z_{uj}) = \mu(Z_{vj}) = 0$, $j \leq n$, $j \neq w$, i.e. $\mu(Z_{uj}) = \mu(Z_{vj})$, $j \leq n$. Therefore,$H(D|C) = H(D|B(L))$. Furthermore, there exists a set $B(S) \subseteq B(L)$, which satisfies $\forall a \in B(S)$, $POS_{B(S)}(D) \neq POS_{B(S)-\{a\}}(D)$. From Definition 2.1, $B(S)$ is a Shannon's entropy reduct. □

Theorem 3.5 shows that there exists a subset of its Liang's entropy reducts which is its Shannon's entropy reduct.

Corollary 3.6: *Let $S = (U, C \cup D)$ be a decision table, $RED_D^L(C)$ a set of all Liang's entropy reducts, and $RED_D^S(C)$ a set of all Shannon entropy reducts, then $min\{|B(S)| : B(S) \in RED_D^S(C)\} \leq min\{|B(L)| : B(L) \in RED_D^L(C)\}$.*

From Corollary 3.6, we can see that, for a decision table, there exists a subset of its Shannon's reducts which is its positive-region reduct.

Corollary 3.7: *Let $S = (U, C \cup D)$ be a decision table. If $B(L)$ is a Liang's entropy reduct, then there exist a positive-region reduct $B(P)$ and a Shannon's entropy reduct $B(S)$ such that $B(P) \subseteq B(S) \subseteq B(L)$.*

The relationship among positive region reducts, Shannon's entropy reducts and Liang's entropy reducts is indicated by Corollary 3.7.

Corollary 3.8: *Let $S = (U, C \cup D)$ be a decision table, $RED^L_D(C)$ a set of all Liang's entropy reducts, $RED^S_D(C)$ a set of all Shannon entropy reducts, and $RED^P_D(C)$ a set of all Shannon entropy reducts, then $min\{|B(P)| : B(P) \in RED^P_D(C)\} \leq min\{|B(S)| : B(S) \in RED^S_D(C)\} \leq min\{|B(L)| : B(L) \in RED^L_D(C)\}$.*

These relationships among the above three kinds of attribute reductions in Corollary 3.8 are illustrated by the following Example 3.1.

Example 3.1 We employ Table 1 to illustrate the relationship among the three kinds of attribute reductions. By computing, we have that
$RED^P_D(C) = \{\{Headache\}, \{Animal\ heat,\ Cough\}\}$,
$RED^S_D(C) = \{\{Headache,\ Muscle\ pain\}, \{Animal\ heat,\ Cough\}\}$, and
$RED^L_D(C) = \{\{Headache,\ Animal\ heat,\ Muscle\ pain\}, \{Animal\ heat,\ Cough\}\}$.
Obviously, one can obtain the following inclusion relationships.

$$\underbrace{\{Headache\}}_{a\ positive-region\ reduct} \subseteq \underbrace{\{Headache,\ Muscle\ pain\}}_{a\ Shannon's\ entropy\ reduct} \subseteq \underbrace{\{Headache,\ Animal\ heat,\ Muscle\ Pain\}}_{a\ Liang's\ entropy\ reduct}.$$

$$\underbrace{\{Animal\ heat,\ Cough\}}_{a\ positive-region\ reduct} \subseteq \underbrace{\{Animal\ heat,\ Cough\}}_{a\ Shannon's\ entropy\ reduct} \subseteq \underbrace{\{Animal\ heat,\ Cough\}}_{a\ Liang's\ entropy\ reduct}.$$

Therefore, $min\{|B(P)| : B(P) \in RED^P_D(C)\} = |\{Headache\}|$, $min\{|B(S)| : B(S) \in RED^S_D(C)\} = |\{Animal\ heat,\ Cough\}|$, $min\{|B(L)| : B(L) \in RED^L_D(C)\} = |\{Animal\ heat,\ Cough\}|$, then, $|\{Headache\}| \leq |\{Animal\ heat,\ Cough\}| \leq |\{Animal\ heat, Cough\}|$.

From the example, we can see that the relationship among minimal positive-region reduct, the minimal Shannon's entropy reduct and the minimal Liang's entropy reduct corresponds to Corollary 3.8.

4. Change mechanism of decision performance of a decision table

In this section, we investigate the change mechanism of decision performance of a decision table from the viewpoint of decision evaluation.

Approximation accuracy of a classification $a_C(F)$ was introduced in (Pawlak 1991). Let $F = \{Y_1, Y_2, \cdots, Y_n\}$ be a classification of the universe U, and C a condition attribute set. Then, C-lower and C-upper approximations of F are given by $\underline{C}F = \{\underline{C}Y_1, \underline{C}Y_2, \cdots, \underline{C}Y_n\}$ and $\overline{C}F = \{\overline{C}Y_1, \overline{C}Y_2, \cdots, \overline{C}Y_n\}$, respectively, where $\underline{C}Y_i = \bigcup\{x \in U \mid [x]_C \subseteq Y_i \in F\}$, $1 \leq i \leq n$, and $\overline{C}Y_i = \bigcup\{x \in U \mid [x]_C \cap Y_i \neq \emptyset, Y_i \in F\}$, $1 \leq i \leq n$. The approximation accuracy of F by C is defined as $a_C(F) = \frac{\sum_{Y_i \in F} |\underline{C}Y_i|}{\sum_{Y_i \in F} |\overline{C}Y_i|}$. The approximation accuracy expresses the percentage of possible correct decisions when classifying objects by employing the attribute set C. In a broad sense, $a_C(F)$ can be used to measure the certainty of a decision table. However, it has some limitations. In (Qian *et al.* 2008b), a new certainty measure α was proposed for overcoming these limitations, which is shown as follows.

Definition 4.1: (Qian *et al.* 2008b) Let $S = (U, C \cup D)$ be a decision table, and $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \to des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The certainty measure α of S is defined as

$$\alpha(S) = \sum_{i=1}^{m} \sum_{j=1}^{n} s(Z_{ij})\mu(Z_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U||X_i|}. \tag{6}$$

Through using the definition, one can get the following Theorem 4.2.

Theorem 4.2: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$, then $\alpha(S') \leq \alpha(S)$, especially, $\alpha(S') = \alpha(S)$ iff $\mu(Z_{uj}) = \mu(Z_{vj})$ for $j \leq n$.*

Proof: From the definition of certainty measure α, it follows that

$$\begin{aligned}
\alpha_\Delta &= \alpha(S') - \alpha(S) \\
&= \sum_{i=1, i\neq u, i\neq v}^{m} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U||X_i|} + \sum_{j=1}^{n} \frac{((|X_u \cup X_v) \cap Y_j|)^2}{|U|(|X_u \cup X_v|)} \\
&\quad - \sum_{i=1, i\neq u, i\neq v}^{m} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U||X_i|} - \sum_{j=1}^{n} \frac{|X_u \cap Y_j|^2}{|U||X_u|} - \sum_{j=1}^{n} \frac{|X_v \cap Y_j|^2}{|U||X_v|} \\
&= \sum_{j=1}^{n} \frac{(|X_u \cap Y_j| + |X_v \cap Y_j|)^2}{|U|(|X_u| + |X_v|)} - \sum_{j=1}^{n} \frac{|X_u \cap Y_j|^2}{|U||X_u|} - \sum_{j=1}^{n} \frac{|X_v \cap Y_j|^2}{|U||X_v|} \\
&= -\sum_{j=1}^{n} \frac{(|X_u||X_v \cap Y_j| - |X_v||X_u \cap Y_j|)^2}{|U||X_u||X_v|(|X_u| + |X_v|)} \\
&= -\sum_{j=1}^{n} \frac{|X_u||X_v|(\mu(Z_{uj}) - \mu(Z_{vj}))^2}{|U|(|X_u| + |X_v|)} \leq 0.
\end{aligned}$$

Clearly, one has that $\alpha_\Delta = 0$ when $\mu(Z_{uj}) = \mu(Z_{vj})$. That is $\alpha(S) = \alpha(S')$. □

The following Corollary 4.3 is directly derived from Theorem 4.2.

Corollary 4.3: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If $U/B \succ U/C$, then $\alpha(S') \leq \alpha(S)$.*

Corollary 4.3 shows that certainty measure α of the decision table after condition attribute set becoming coarser will be not more than the one of the original table.

As follows, we analyze the change mechanism of the consistency measure of a decision table. The consistency measure from (Qian *et al.* 2008b) is another important measure for assessing the decision performance of a decision table, which is shown in Definition 4.4.

Definition 4.4: (Qian *et al.* 2008b) Let $S = (U, C \cup D)$ be a decision table and

$RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The consistency measure β of S is defined as

$$\beta(S) = \sum_{i=1}^{m} \frac{|X_i|}{|U|}[1 - \frac{4}{|X_i|}\sum_{j=1}^{N_i}|X_i \cap Y_j|\mu(Z_{ij})(1-\mu(Z_{ij}))], \tag{7}$$

where N_i is the number of decision rules determined by the condition class X_i, and $\mu(Z_{ij})$ is the certainty degree of the rule Z_{ij}.

Using the consistency measure, the following Theorem 4.5 can be derived.

Theorem 4.5: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, if $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$, $U/D = \{Y_1, Y_2, \cdots, Y_n\}$ then the relationship between $\beta(S')$ and $\beta(S)$ is uncertain, especially, $\beta(S) = \beta(S')$ iff $\mu(Z_{uj}) = \mu(Z_{vj})$ for $j \leq n$.*

Proof: From the definition of consistency measure, it is easy to know that

$$\begin{aligned}\beta(S) &= \sum_{i=1}^{m} \frac{|X_i|}{|U|}[1 - \frac{4}{|X_i|}\sum_{j=1}^{N_i}|X_i \cap Y_j|\mu(Z_{ij})(1-\mu(Z_{ij}))] \\ &= 1 - \frac{4}{|U|}\sum_{i=1}^{m}\sum_{j=1}^{n}|X_i \cap Y_j|\mu(Z_{ij})(1-\mu(Z_{ij}))\end{aligned}$$

$$\begin{aligned}\beta_\Delta &= \beta(S') - \beta(S) \\ &= \frac{4}{|U|}\sum_{j=1}^{n}\left(\frac{|X_u \cap Y_j|^2}{|X_u|} - \frac{|X_u \cap Y_j|^3}{|X_u|^2}\right) + \frac{4}{|U|}\sum_{j=1}^{n}\left(\frac{|X_v \cap Y_j|^2}{|X_v|} - \frac{|X_v \cap Y_j|^3}{|X_v|^2}\right) \\ &\quad - \frac{4}{|U|}\sum_{j=1}^{n}\left(\frac{(|X_u \cap Y_j| + |X_v \cap Y_j|)^2}{|X_u| + |X_v|} - \frac{(|X_u \cap Y_j| + |X_v \cap Y_j|)^3}{(|X_u| + |X_v|)^2}\right).\end{aligned}$$

Let $x = |X_u|$, $y = |X_v|$, $\delta_j = \frac{|X_u \cap Y_j|}{|X_u|}$ and $\sigma_j = \frac{|X_v \cap Y_j|}{|X_v|}$. It follows that

$$\begin{aligned}\beta_\Delta &= \sum_{j=1}^{n}\frac{(\delta_j x)^2}{x} - \frac{(\delta_j x)^3}{x^2} + \sum_{j=1}^{n}\frac{(\sigma_j y)^2}{y} - \frac{(\sigma_j y)^3}{y^2} \\ &\quad - \sum_{j=1}^{n}\frac{(\delta_j x + \sigma_j y)^2}{x+y} - \frac{(\delta_j x + \sigma_j y)^3}{(x+y)^2}) \\ &= \sum_{j=1}^{n}\frac{xy}{(x+y)^2}\left(((\delta_j - \sigma_j)^2 - 2\delta_j^3 - \sigma_j^3 + 3\delta_j^2\sigma_j)x\right) \\ &\quad + \sum_{j=1}^{n}\frac{xy}{(x+y)^2}\left(((\delta_j - \sigma_j)^2 - \delta_j^3 - 2\sigma_j^3 + 3\delta_j\sigma_j^2)y\right) \\ &= \sum_{j=1}^{n}\frac{xy(\delta_j - \sigma_j)^2}{(x+y)^2}\left((1 - 2\delta_j - \sigma_j)x + (1 - 2\sigma_j - \delta_j)y\right).\end{aligned}$$

Obviously, when $\delta_j = \sigma_j$, $\forall j \le n$, i.e.$\mu(Z_{uj}) = \mu(Z_{vj})$, we have that $\beta_\Delta = 0$. Thus, $\beta(S') = \beta(S)$. Otherwise, the value of β_Δ is uncertain. □

Theorem 4.5 easily deduces the following corollary.

Corollary 4.6: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, if $U/C \prec U/B$, then the relationship between $\beta(S')$ and $\beta(S)$ is uncertain.*

Corollary 4.6 indicates that, for a decision table, the change of the consistency measure β is uncertain after the condition attribute set becoming coarser.

Theorem 4.7: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, if $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$, $U/D = \{Y_1, Y_2\}$, then $\beta(S') \le \beta(S)$, especially, $\beta(S) = \beta(S')$ iff $\mu(Z_{uj}) = \mu(Z_{vj})$ for $j \le 2$.*

Proof: Let $x = |X_u|$, $y = |X_v|$, $\delta_j = \frac{|X_u \cap Y_j|}{|X_u|}$ and $\sigma_j = \frac{|X_v \cap Y_j|}{|X_v|}$. From the proof of Theorem 4.5, we have that

$$\begin{aligned}\beta_\Delta &= \beta(S') - \beta(S)\\ &= \sum_{j=1}^{n} \frac{xy(\delta_j - \sigma_j)^2}{(x+y)^2}\left((1 - 2\delta_j - \sigma_j)x + (1 - 2\sigma_j - \delta_j)y\right).\end{aligned}$$

Furthermore, by the existing condition $U/D = \{Y_1, Y_2\}$, we have that $\delta_1 + \delta_2 = 1$ and $\sigma_1 + \sigma_2 = 1$. Thus, it follows that

$$\begin{aligned}\beta_\Delta &= \frac{xy(\delta_1 - \sigma_1)^2}{(x+y)^2}\left((1 - 2\delta_1 - \sigma_1)x + (1 - 2\sigma_1 - \delta_1)y\right)\\ &+ \frac{xy(\delta_2 - \sigma_2)^2}{(x+y)^2}\left((1 - 2\delta_2 - \sigma_2)x + (1 - 2\sigma_2 - \delta_2)y\right)\\ &= -2\frac{xy(\delta_1 - \sigma_1)^2}{(x+y)} \le 0.\end{aligned}$$

Obviously, when $\delta_j = \sigma_j$ i.e.$\mu(Z_{uj}) = \mu(Z_{vj})$ for $j \le 2$, we have that $\beta(S') = \beta(S)$. □

The following Corollary 4.8 generalizes the results of Theorem 4.7.

Corollary 4.8: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, if $U/C \prec U/B$ and $U/D = \{Y_1, Y_2\}$, then $\beta(S') \le \beta(S)$.*

Corollary 4.8 shows that, for a decision table with two decision values, consistency measure β of the decision will be not more than its one after condition attribute set becoming coarser.

In Literature (Qian *et al.* 2008b), support measure of a decision table is proposed for computing entire support measure of all decision rules. In the following, we will consider the mechanism of the measure.

Definition 4.9: (Qian *et al.* 2008b) Let $S = (U, C \cup D)$ be a decision table and $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The support measure γ of S is defined as

$$\gamma(S) = \sum_{i=1}^{m}\sum_{j=1}^{n} s^2(Z_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U|^2}. \quad (8)$$

The following Theorem 4.10 gives the monotonicity of the support measure in the context of decision tables.

Theorem 4.10: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, if $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}$, $U/D = \{Y_1, Y_2, \cdots, Y_n\}$, $X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$, then $\gamma(S') \geq \gamma(S)$.*

Proof: By the existing condition, it follows that

$$\begin{aligned}\gamma_\Delta &= \gamma(S') - \gamma(S) \\ &= \sum_{k=1}^{l}\sum_{j=1}^{n} \frac{|X'_k \cap Y_j|^2}{|U|^2} - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U|^2} \\ &= \sum_{j=1}^{n} \frac{(|X_u \cap Y_j| + |X_v \cap Y_j|)^2}{|U|^2} - \sum_{j=1}^{n} \frac{|X_u \cap Y_j|^2}{|U|^2} - \sum_{j=1}^{n} \frac{|X_v \cap Y_j|^2}{|U|^2} \\ &= \sum_{j=1}^{n} \frac{2|X_u \cap Y_j||X_v \cap Y_j|}{|U|^2} \geq 0.\end{aligned}$$

□

Corollary 4.11: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If $U/C \prec U/B$, then $\gamma(S') \geq \gamma(S)$.*

From Corollary 4.11, we know that, for a given decision, the finer condition attribute set usually decreases the support measure γ.

5. Change of decision performance induced by reduction approaches

In this section, we investigate the three kinds of attribute reduction mthods, namely positive region reduction, Shannon's entropy reduction and Liang's entropy reduction. We analyze the difference between the decision performance of a reduced decision table and that of the original one.

5.1 *Change of decision performance induced by positive-region reduction*

The analysis on change of decision performance of a decision table after performing a positive-region reduction will be shown in this subsection. They can be discovered by the following six theorems and three examples.

Theorem 5.1: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, and B be a positive-region reduct of C. If $X_u \in U/C$ and $X_v \in U/C$ are in the consistent part of the decision table S, and $X_u \cup X_v = X_w$, $X_w \in U/B$, then $\mu(Z_{uj}) = \mu(Z_{vj})$ for $Y_j \in U/D$, where $\mu(Z_{ij}) = |X_i \cap Y_j|/|X_i|$.*

Proof: From the condition, we have that the two classes X_u and X_v in the consistent part of S combines a new condition class X_w in S'. Therefore, the condition classes X_w will fall in the inconsistent part of S' if $\exists j \leq n$ such that $\mu(Z_{uj}) \neq \mu(Z_{vj})$. Clearly, the positive region of S is unequal to the one of S', which is in contradictive with the assumption that B is a positive-region reduct of C. Thus, $\mu(Z_{uj}) = \mu(Z_{vj})$ for $Y_j \in U/D$. □

Theorem 5.1 indicates that if some condition classes in the consistent part of a decision table combine to a new condition class after performing the positive-region reduction, then the rules induced by these condition classes have the same certainty measures.

Moreover, we first investigate the change mechanism of entire certainty measure α with respect to the positive-region reduction.

Theorem 5.2: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If S is consistent and B is a positive-region reduct of C, then*

$$\alpha(S') = \alpha(S), \beta(S') = \beta(S), \gamma(S') \geq \gamma(S).$$

Proof: By the existing condition that B is a positive-region reduct of C, we have $U/B \succeq U/C$. It is obviously that $\alpha(S') = \alpha(S), \beta(S') = \beta(S), \gamma(S') = \gamma(S)$ if $U/B = U/C$, and the case $U/B \succ U/C$ will be analyzed in detail.

For simplicity, without any loss of generality, let $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$.

Since the decision table S is consistent, then the decision table S' after performing the positive-region reduction is also consistent. Furthermore, according to Theorem 5.1, we have that $\mu(Z_{uj}) = \mu(Z_{vj})$ for $j \leq n$. Thus, by Theorem 4.2, Theorem 4.5 and Theorem4.10, we have that $\alpha(S') = \alpha(S), \beta(S') = \beta(S), \gamma(S') \geq \gamma(S)$, respectively. □

Theorem 5.2 shows that, for a consistent table, if it is reduced by performing the positive-region reduction, then the certainty measure of the table will be unchanged.

Theorem 5.3: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If B is a positive-region reduct of C, then $\alpha(S') \leq \alpha(S)$, the relationship between $\beta(S')$ and $\beta(S)$ is uncertain, and $\gamma(S') \geq \gamma(S)$.*

Proof: From the condition that B is a positive-region reduct of C, we have $U/B \succeq U/C$. It is obviously that $\alpha(S') = \alpha(S), \beta(S') = \beta(S), \gamma(S') = \gamma(S)$ if $U/B = U/C$, and the case $U/B \succ U/C$ will be analyzed in detail.

For simplicity, without any loss of generality, we suppose that $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$.

Through using the positive-region reduction, the change of condition classes has two cases. One is combination of the condition classes in the consistent part of a decision table, the other is combination of the condition classes in the inconsistent part of a decision table. These two cases are listed as follows.

1) The condition classes combined in the consistent part

Let the two classes X_u and X_v in the consistent part of decision table S become a new condition class $X_u \cup X_v$ after performing the positive-region reduction and the other condition classes are unchanged. From Theorem 5.1, it follows that $\mu(Z_{uj}) = \mu(Z_{vj})$, $j \leq n$. Furthrermore, by Theorem 4.2, Theorem 4.5 and Theorem 4.10, one has that $\alpha(S') = \alpha(S)$, $\beta(S') = \beta(S)$ and $\gamma(S') \geq \gamma(S)$, respectively.

2) The condition classes combined in the inconsistent part

Let the two classes X_u and X_v in the inconsistent part of table S are combined to a class $X_u \cup X_v$ after the positive-region reduction and other condition classes are unchanged. From Theorem 4.2, Theorem 4.5 and Theorem 4.10, it follows that $\alpha(S') \leq \alpha(S)$, the relationship between $\beta(S')$ and $\beta(S)$ is uncertain, and $\gamma(S') \geq \gamma(S)$.

In conclusion, $\alpha(S') \leq \alpha(S)$, the relationship between $\beta(S')$ and $\beta(S)$ is uncertain, and $\gamma(S') \geq \gamma(S)$, if B is a positive-region reduct of C. □

Theorem 5.3 states that, for a decision table, the certainty measure α of the decision table after using the positive-region reduction will be no more than that of original one, the consistency measure β of the reduced table will be uncertain after performing a positive-region reduction, and the support measure γ after using the positive-region reduction will be no less than that of original one. Example 5.1 shows their change mechanism.

Example 5.1 We employ Table 2 and Table 3 to illustrate the change of the decision performance of a decision table after performing the positive-region reduction.

It is easy to calculate using Definition 2.1 that the set of all positive-region reducts of Table 2 is $RED_D^P(C) = \{\{Muscle\ pain,\ Cough\}\}$. Let $B(P) = \{Muscle\ pain,\ Cough\}$, $S' = (U, B(P) \cup D)$, we have that

$$\alpha(S) = 0.4861,\ \alpha(S') = 0.4833,$$
$$\beta(S) = 0.2639,\ \beta(S') = 0.2667,$$
$$\gamma(S) = 0.1528,\ \gamma(S') = 0.2778.$$

It is clear that

$$\alpha(S') < \alpha(S), \beta(S') > \beta(S), \gamma(S') > \gamma(S).$$

Furthermore, we can obtain that the set of all positive-region reducts of Table 3 is $RED_D^P(C) = \{\{Muscle\ pain,\ Cough\}\}$. Let $B(P) = \{Muscle\ pain,\ Cough\}$, $S' = (U, B(P) \cup D)$, one has

$$\alpha(S) = 0.4861,\ \alpha(S') = 0.4860,$$
$$\beta(S) = 0.2639,\ \beta(S') = 0.2600,$$
$$\gamma(S) = 0.1528,\ \gamma(S') = 0.2500.$$

Obviously,

$$\alpha(S') < \alpha(S), \beta(S') < \beta(S), \gamma(S') > \gamma(S).$$

Theorem 5.4: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables, $U/D = \{Y_1, Y_2\}$. If B is a positive-region reduct of C, then $\alpha(S') \leq \alpha(S)$, $\beta(S') \leq \beta(S)$ and $\gamma(S') \geq \gamma(S)$.*

Proof: According to the condition that B is a positive-region reduct of C, we have $U/B \succeq U/C$. It is obviously that $\alpha(S') = \alpha(S), \beta(S') = \beta(S), \gamma(S') = \gamma(S)$ if $U/B = U/C$, and the case $U/B \succ U/C$ will be investigated in detail.

For simplicity, without any loss of generality, we suppose that $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m, X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2\}$.

Through using the positive-region reduction, the change of condition classes has two cases. One is combination of the condition classes in the consistent part of a decision table, the other is combination of the condition classes in the inconsistent part of a decision table. These two cases are listed as follows.

1) The condition classes combined in the consistent part

Let the two classes X_u and X_v in the consistent part of decision table S become a new condition class $X_u \cup X_v$ after performing the positive-region reduction and the other condition classes are unchanged. From Theorem 5.1, it follows that $\mu(Z_{uj}) = \mu(Z_{vj})$, $j \leq n$. Furthrermore, by Theorem 4.2, Theorem 4.7 and Theorem 4.10, one has that $\alpha(S') = \alpha(S)$, $\beta(S') = \beta(S)$ and $\gamma(S') \geq \gamma(S)$, respectively.

2) The condition classes combined in the inconsistent part

Let the two classes X_u and X_v in the inconsistent part of table S are combined to a class $X_u \cup X_v$ after the positive-region reduction and other condition classes are unchanged. From Theorem 4.2, Theorem 4.7 and Theorem 4.10, it follows that $\alpha(S') \leq \alpha(S)$, $\beta(S') \leq \beta(S)$ and $\gamma(S') \geq \gamma(S)$.

In conclusion, $\alpha(S') \leq \alpha(S)$, $\beta(S') \leq \beta(S)$ and $\gamma(S') \geq \gamma(S)$, if B is a positive-region reduct of C and there are only two decision values in a decision table. □

Theorem 5.3 states that, for a decision table with two decision values, the certainty measure α and the consistence measure β after using the positive-region reduction will be no more than that of original table, and the support measure γ will be no less than that of original one. It is illustrated by the following example.

Example 5.2 We employ Table 1 to illustrate the change of the decision performance of a decision table after performing the positive-region reduction.

It is easy to obtain by Definition 2.1 that the set of all positive-region reducts $RED_D^P(C) = \{\{Headache\}, \{Animal\ heat,\ Cough\}\}$. Let $B_1(P) = \{Headache\}$, $B_2(P) = \{Animal\ heat,\ Cough\}$, $S_1 = (U, B_1(P) \cup D)$ and $S_2 = (U, B_2(P) \cup D)$, we have that

$$\alpha(S) = 0.5667,\ \alpha(S_1) = 0.5556,\ \alpha(S_2) = 0.5667,$$
$$\beta(S) = 0.1333,\ \beta(S_1) = 0.1111,\ \beta(S_2) = 0.1333,$$
$$\gamma(S) = 0.1200,\ \gamma(S_1) = 0.4200,\ \gamma(S_2) = 0.1200.$$

It is clear that

$$\alpha(S_1) < \alpha(S),\ \alpha(S_2) = \alpha(S),$$
$$\beta(S_1) < \beta(S),\ \beta(S_2) = \beta(S),$$
$$\gamma(S_1) > \gamma(S),\ \gamma(S_2) = \gamma(S).$$

5.2 *Change of decision performance induced by Shannon's entropy reduction*

In this subsection, we will analyze the change mechanism of decision performance of a decision table through performing Shannon's entropy reduction.

Theorem 5.5: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. If B is a Shannon's entropy reduct of C, then*

$$\alpha(S) = \alpha(S'), \beta(S) = \beta(S') \text{ and } \gamma(S) \leq \gamma(S').$$

Proof: For simplicity, without any loss of generality, let $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1}, X_{v+1}, \cdots, X_m,$

$X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$. From the existing condition that B is a Shannon's entropy reduct of C and Theorem 3.1, it follows that $\mu(Z_{uj}) = \mu(Z_{vj})$.

Therefore, by Theorem 4.2, $\alpha(S) = \alpha(S')$, from Theorem 4.5, $\beta(S) = \beta(S')$, and according to Theorem 4.7, $\gamma(S) \leq \gamma(S')$. □

Theorem 5.5 states that the certainty measure α of a decision table will be unchangeable after Shannon's entropy reduction, the consistent measure β of a decision table will also be unchangeable after using Shannon's entropy reduction, and the support measure γ through using Shannon's entropy reduction will be no less than that of original one. Example 5.4 illustrates the change mechanism of the support measure.

Example 5.3 We employ Table 1 to illustrate the change of the decision performance of a decision table after performing the Shannon's entropy reduction.

It is easy to calculate from Definition 2.2 that the set of all Shannon's entropy reducts $RED_D^S(C) = \{\{Headache,\ Muscle\ pain\}, \{Animal\ heat,\ Cough\}\}$. Let $B_1(S) = \{Headache,\ Muscle\ pain\}$, $B_2(S) = \{Animal\ heat,\ Cough\}$ and $S_1 = (U, B_1(S) \cup D)$, $S_2 = (U, B_2(S) \cup D)$, we have that

$$\alpha(S) = 0.5667,\ \alpha(S_1) = 0.5667,\ \alpha(S_2) = 0.5667,$$
$$\beta(S) = 0.1333,\ \beta(S_1) = 0.1333,\ \beta(S_2) = 0.1333,$$
$$\gamma(S) = 0.1200,\ \gamma(S_1) = 0.2400,\ \gamma(S_2) = 0.1200.$$

Obviously,

$$\alpha(S_1) = \alpha(S_2) = \alpha(S), \beta(S_1) = \beta(S_2) = \beta(S), \gamma(S_1) > \gamma(S), \gamma(S_2) = \gamma(S).$$

5.3 *Change of decision performance induced by Liang's entropy reduction*

In this subsection, one will express the variety of decision performance through Liang's entropy reduction.

Theorem 5.6: *Let $S = (U, C \cup D)$ and $S' = (U, B \cup D)$ be two decision tables. if B is a Liang's entropy reduct of C, then*

$$\alpha(S) = \alpha(S'), \beta(S) = \beta(S') \text{ and } \gamma(S) \leq \gamma(S').$$

Proof: For simplicity, without any loss of generality, let $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/B = \{X_1, X_2, \cdots, X_{u-1}, X_{u+1}, \cdots, X_{v-1},\ X_{v+1}, \cdots, X_m,$ $X_u \cup X_v\}$ and $U/D = \{Y_1, Y_2, \cdots, Y_n\}$. Through using Liang's entropy reduction, we suppose that the two classes X_u and $X_v (u, v < m)$ combine to a new class $X_u \cup X_v$. From Theorem 3.4 it follows that $\mu(Z_{uj}) = \mu(Z_{vj})$.

Therefore, from Theorem 4.2, $\alpha(S) = \alpha(S')$, by Theorem 4.5, $\beta(S) = \beta(S')$, and from Theorem 4.7, $\gamma(S) \leq \gamma(S')$. □

Theorem 5.6 shows that the certainty measure α will be unchanged after performing Liang's entropy reduction, the consistency measure will also be unchanged after performing a Liang's entropy reduction, and the support measure after using Liang's entropy reduction will be no less than that of original one. This idea can be explained by the following example.

Example 5.4 We employ Table 1 to illustrate the change of the decision performance of a decision table after performing Liang's entropy reduction.

It is easy to get using Definition 2.3 that the set of all Liang's entropy reducts $RED_D^L(C) = \{\{Headache,\ Animalheat,\ Musclepain\}, \{Animal\ heat,\ Cough\}\}$. Let $B_1(L) = \{Headache,\ Animal\ heat,\ Musclepain\}$, $B_2(L) = \{Animal\ heat,\ Cough\}$ and $S_1 = (U, B_1(L) \cup D)$, $S_2 = (U, B_2(L) \cup D)$. we have that

$$\alpha(S) = 0.5667,\ \alpha(S_1) = 0.5667,\ \alpha(S_2) = 0.5667,$$
$$\beta(S) = 0.1333,\ \beta(S_1) = 0.1333,\ \beta(S_2) = 0.1333,$$
$$\gamma(S) = 0.1200,\ \gamma(S_1) = 0.1200,\ \gamma(S_2) = 0.1200.$$

Obviously,

$$\alpha(S_1) = \alpha(S_2) = \alpha(S), \beta(S_1) = \beta(S_2) = \beta(S), \gamma(S_1) = \gamma(S_2) = \gamma(S).$$

6. Example analysis

In this section, through experimental analysis, we illustrate the change of the decision performance after using the positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction, for general decision tables. We have download the data set Spect from UCI database (Spect is a decision table with two decision values). In order to verify their performance, we randomly extract 150 objects from the data set 100 times. As the limitation of the paper's length, one of 100 tables extracted from Spect is selected to verify our results.

6.1 *Performance change deriving from positive-region redction*

All the positive-region reducts and their corresponding three performance measures of the original table are shown in Table 4 and Figures 1-3. The values of α, β and γ of the original table and the corresponding reduced tables are shown in Table 4. Figures 1-3 express that the value of each of α, β and γ with respect to every positive-region reduct respectively.

From Table 4 and Figures 1-3, it is easy to draw the following conclusion. Through using a positive-region reduction, the certainty measure α and the consistency measure β are not bigger than the original certainty measure and the original consistency measure respectively, and the support measure γ is not smaller than the original support measure.

6.2 *Performance change deriving from Shannon's entropy reduction*

All the Shannon's entropy reducts and their corresponding three performance measures of the original table are shown in Table 5 and Figures 4-6.The values of α, β and γ of the original table and the corresponding reduced tables are shown in Table 5. Figures 4-6 express that the value of each of α, β and γ as to every Shannon's entropy reduct respectively.

From Table 5 and Figures 4-6, it is easy to draw the following conclusion: after performing a Shannon's entropy reduction, each of the certainty measure α and the consistency measure β is the same as each of those induced by a original decision table, and the support measure γ is not smaller than the original support measure.

6.3 *Performance change deriving from Liang's entropy reduction*

All the Liang's entropy reducts and their corresponding three performance measures of the original table are shown in Table 6 and Figures 7-9. The values of α, β and γ of the original table and the corresponding reduced tables are appeared in Table 6. Figures 7-9 express that the value of each of α, β and γ with respect to every positive-region reduct respectively.

From Table 6 and Figures 7-9, it is easy to draw the following conclusions: after performing a Liang's entropy reduction, the change is similar to Shannon's entropy reduction, each of the certainty measure α and the consistency measure β is the same as each of those induced by a original decision table, and the support measure γ is not smaller than the original support measure.

7. Conclusions

Certainty measure, consistency measure and support measure are three important measures for evaluating the decision performance of a decision table. In this paper, we have analyzed the change mechanism of the decision performance after performing the positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction, and have obtained some of their important properties. These three measures may be changed through using a positive-region reduction. However, the certainty measure and the consistency measure are unchanged after using a Shannon's entropy reduction and Liang's entropy reduction, and the support measure is usually increased. These results may be helpful for determining which of the positive-region reduction, Shannon's entropy reduction and Liang's entropy reduction is preferred for a practical decision problem in the context of complete decision tables. Further development will be focus on change mechanism of three evaluation measures in the context of incomplete decision tables.

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Table 1. A decision table about diagnosing rheum

Patients	Headache	Muscle pain	Animal heat	Cough	Rheum
e_1	Yes	Yes	Normal	No	No
e_2	Yes	Yes	High	No	No
e_3	Yes	Yes	Normal	No	Yes
e_4	Yes	Yes	high	No	Yes
e_5	Yes	No	High	Yes	Yes
e_6	Yes	No	High	Yes	Yes
e_7	Yes	No	High	Yes	No
e_8	Yes	Yes	Very high	Yes	Yes
e_9	Yes	Yes	Very high	Yes	No
e_{10}	No	Yes	Normal	Yes	Yes

Table 2. A decision table about diagnosing rheum

Patients	Headache	Muscle pain	Animal heat	Cough	Rheum
e_1	Yes	No	high	Yes	No
e_2	Yes	Yes	high	No	Yes
e_3	Yes	Yes	Normal	Yes	No
e_4	Yes	Yes	Normal	Yes	No
e_5	Yes	Yes	Normal	Yes	Yes
e_6	Yes	Yes	Normal	Yes	Possible
e_7	No	Yes	High	Yes	No
e_8	No	Yes	High	Yes	No
e_9	No	Yes	High	Yes	No
e_{10}	No	Yes	High	Yes	Yes
e_{11}	No	Yes	High	Yes	Yes
e_{12}	No	Yes	High	Yes	Possible

Table 3. A decision table about diagnosing rheum

Patients	Headache	Muscle pain	Animal heat	Cough	Rheum
e_1	Yes	No	high	Yes	No
e_2	Yes	Yes	high	No	Yes
e_3	Yes	Yes	Normal	Yes	No
e_4	Yes	Yes	Normal	Yes	No
e_5	Yes	Yes	Normal	Yes	Yes
e_6	Yes	Yes	Normal	Yes	Possible
e_7	No	Yes	High	Yes	No
e_8	No	Yes	High	Yes	Yes
e_9	No	Yes	High	Yes	Yes
e_{10}	No	Yes	High	Yes	Yes
e_{11}	No	Yes	High	Yes	Possible
e_{12}	No	Yes	High	Yes	Possible

Table 4. All positive-region reducts and decision performance measures of corresponding reduced tables

No.	Reducts	α	β	γ
1	1,2,3,4,8,9,10,11,12,14,15,16,19,20,21	0.90379	0.80758	0.01396
2	1,2,3,4,7,8,9,10,11,13,14,15,16,19,20,21	0.90444	0.80889	0.01244
3	1,2,3,4,8,9,10,11,12,14,17,20,21,22	0.90339	0.80679	0.01102
4	1,2,3,4,8,9,10,11,12,14,19,21,22	0.90381	0.80762	0.01262
5	1,2,3,4,5,8,9,10,11,13,14,17,20,21,22	0.90381	0.80762	0.01058
6	1,2,3,4,7,8,9,10,11,13,14,17,20,21,22	0.90400	0.80800	0.01004
7	1,2,3,4,8,9,10,11,13,14,16,17,20,21,22	0.90381	0.80762	0.01031
8	1,2,3,4,8,9,10,11,13,14,19,20,21,22	0.90444	0.80889	0.01013
9	1,2,3,4,6,8,9,10,12,14,15,16,19,20,21	0.90379	0.80758	0.01387
10	1,2,3,4,5,6,8,9,11,12,14,15,19,20,21	0.90379	0.80758	0.01396
11	1,2,3,4,6,8,9,10,11,12,14,15,19,20,21	0.90379	0.80758	0.01396
12	1,2,3,4,6,8,9,11,12,14,15,17,19,20,21	0.90379	0.80758	0.01378
13	1,2,3,4,6,7,8,9,10,13,14,15,16,19,20,21	0.90444	0.80889	0.01236
14	1,2,3,4,5,6,7,8,9,11,13,14,15,19,20,21	0.90444	0.80889	0.01262
15	1,2,3,4,6,7,8,9,10,11,13,14,15,19,20,21	0.90444	0.80889	0.01244
16	1,2,3,4,6,7,8,9,11,13,14,15,17,19,20,21	0.90444	0.80889	0.01236
17	1,3,4,6,8,9,10,12,14,16,17,20,21,22	0.90339	0.80679	0.01111
18	1,3,4,6,8,9,10,12,14,16,19,21,22	0.90381	0.80762	0.01253
19	1,3,4,5,6,8,9,11,12,14,17,21,22	0.90337	0.80673	0.01307
20	1,3,4,6,8,9,10,11,12,14,17,21,22	0.90337	0.80673	0.01289
21	1,3,4,6,8,9,11,12,14,17,19,21,22	0.90381	0.80762	0.01262
22	1,3,4,5,6,8,9,11,12,14,19,21,22	0.90381	0.80762	0.01271
23	1,3,4,6,8,9,10,11,12,14,19,21,22	0.90381	0.80762	0.01262
24	1,2,3,4,6,8,9,10,13,14,16,17,20,21,22	0.90381	0.80762	0.01022
25	1,3,4,6,7,8,9,10,13,14,16,17,20,21,22	0.90400	0.80800	0.00987
26	1,2,3,4,6,8,9,10,13,14,16,18,19,20,21,22	0.90444	0.80889	0.00967
27	1,3,4,6,7,8,9,10,13,14,16,19,20,21,22	0.90444	0.80889	0.00960
28	1,2,3,4,5,6,8,9,11,13,14,17,20,21,22	0.90381	0.80762	0.01076
29	1,2,3,4,6,8,9,11,13,14,17,19,20,21,22	0.90444	0.80889	0.01004
30	1,3,4,5,6,7,8,9,11,13,14,17,20,21,22	0.90400	0.80800	0.01013
31	1,3,4,6,7,8,9,11,13,14,16,17,19,20,21,22	0.90444	0.80889	0.00960
32	1,2,3,4,5,6,8,9,11,13,14,19,20,21,22	0.90444	0.80889	0.01033
33	1,3,4,5,6,7,8,9,11,13,14,19,20,21,22	0.90444	0.80889	0.00969
⋆	Original table	0.90444	0.80889	0.00933

Table 5. All Shannon's entropy reducts and decision performance measures of corresponding reduced tables

No.	Reducts	α	β	γ
1	1,2,3,4,8,9,10,11,12,13,14,19,21,22	0.90444	0.80889	0.01120
2	1,2,3,4,7,8,9,10,11,13,14,15,16,19,20,21	0.90444	0.80889	0.01240
3	1,2,3,4,8,9,10,11,12,13,14,15,16,19,20,21	0.90444	0.80889	0.01227
4	1,2,3,4,8,9,10,11,13,14,19,20,21,22	0.90444	0.80889	0.01013
5	1,2,3,4,5,6,7,8,9,11,13,14,15,19,20,21	0.90444	0.80889	0.01262
6	1,2,3,4,6,7,8,9,10,11,13,14,15,19,20,21	0.90444	0.80889	0.01244
7	1,2,3,4,6,7,8,9,11,13,14,15,17,19,20,21	0.90444	0.80889	0.01236
8	1,2,3,4,5,6,8,9,11,12,13,14,15,19,20,21	0.90444	0.80889	0.01236
9	1,2,3,4,6,8,9,10,11,12,13,14,15,19,20,21	0.90444	0.80889	0.01227
10	1,2,3,4,6,8,9,11,12,13,14,15,17,19,20,21	0.90444	0.80889	0.01209
11	1,2,3,4,6,7,8,9,10,13,14,15,16,19,20,21	0.90444	0.80889	0.01236
12	1,2,3,4,6,8,9,10,12,13,14,15,16,19,20,21	0.90444	0.80889	0.01218
13	1,3,4,5,6,8,9,11,12,13,14,19,21,22	0.90444	0.80889	0.01120
14	1,3,4,6,8,9,10,11,12,13,14,19,21,22	0.90444	0.80889	0.01111
15	1,3,4,6,8,9,11,12,13,14,17,19,21,22	0.90444	0.80889	0.01102
16	1,2,3,4,5,6,8,9,11,13,14,19,20,21,22	0.90444	0.80889	0.01013
17	1,2,3,4,6,8,9,11,13,14,17,19,20,21,22	0.90444	0.80889	0.01004
18	1,3,4,5,6,7,8,9,11,13,14,19,20,21,22	0.90444	0.80889	0.00969
19	1,3,4,6,7,8,9,11,13,14,16,17,19,20,21,22	0.90444	0.80889	0.00960
20	1,3,4,6,8,9,10,12,13,14,16,19,21,22	0.90444	0.80889	0.01111
21	1,2,3,4,6,8,9,10,13,14,16,17,19,20,21,22	0.90444	0.80889	0.00960
22	1,2,3,4,6,8,9,10,13,14,16,18,19,20,21,22	0.90444	0.80889	0.00969
23	1,3,4,6,7,8,9,10,13,14,16,19,20,21,22	0.90444	0.80889	0.00960
⋆	Original table	0.90444	0.80889	0.00933

Table 6. All Liang's entropy reducts and decision performance measures of corresponding reduced tables

No.	Reducts	α	β	γ
1	1,2,3,4,7,8,9,10,11,13,14,19,20,21,22	0.90444	0.80889	0.00969
2	1,2,3,4,8,9,10,11,12,13,14,19,20,21,22	0.90444	0.80889	0.00978
3	1,3,4,6,7,8,9,10,13,14,16,19,20,21,22	0.90444	0.80889	0.00960
4	1,3,4,5,6,7,8,9,11,13,14,19,20,21,22	0.90444	0.80889	0.00969
5	1,2,3,4,6,7,8,9,11,13,14,17,19,20,21,22	0.90444	0.80889	0.00969
6	1,3,4,6,7,8,9,11,13,14,16,17,19,20,21,22	0.90444	0.80889	0.00960
7	1,3,4,6,8,9,10,12,13,14,16,19,20,21,22	0.90444	0.80889	0.00978
8	1,3,4,5,6,8,9,11,12,13,14,19,20,21,22	0.90444	0.80889	0.00987
9	1,3,4,6,8,9,10,11,12,13,14,19,20,21,22	0.90444	0.80889	0.00978
10	1,3,4,6,8,9,11,12,13,14,17,19,20,21,22	0.90444	0.80889	0.00969
⋆	Original table	0.90444	0.80889	0.00933

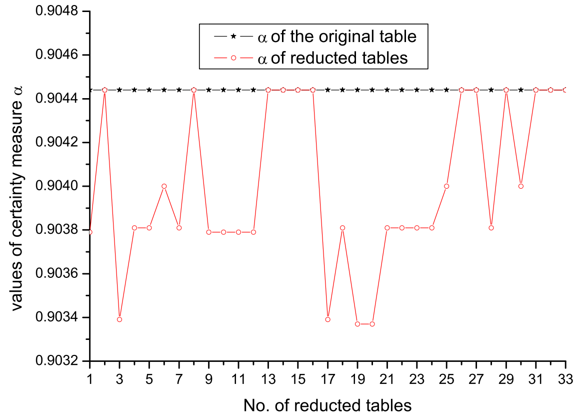

Figure 1. Variation of the certainty measure α after Positive-region reducts.

Figure 2. Variation of the consistency measure β after Positive-region reducts.

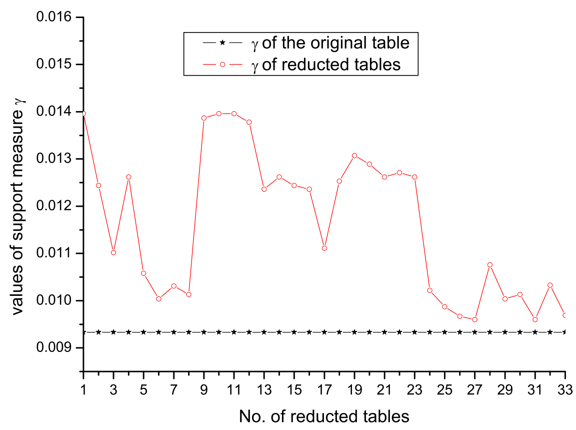

Figure 3. Variation of the support measure γ after Positive-region reducts.

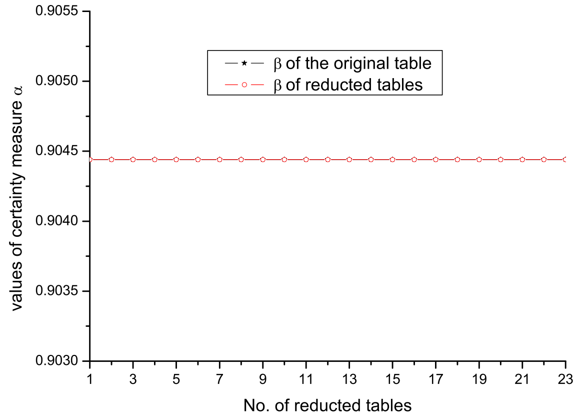

Figure 4. Variation of the certainty measure α after the Shannon's entropy reducts.

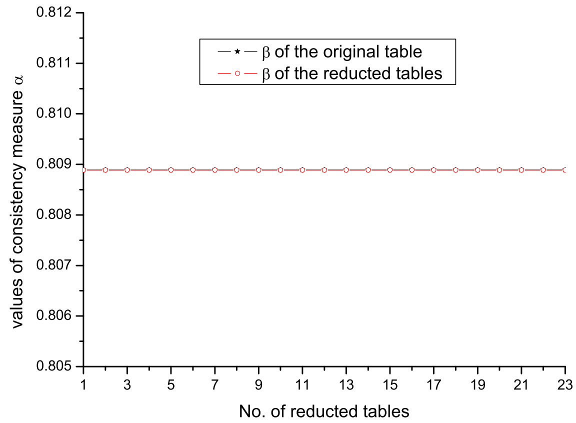

Figure 5. Variation of the consistency measure β after the Shannon's entropy reducts.

Figure 6. Variation of the support measure γ after the Shannon's entropy reducts.

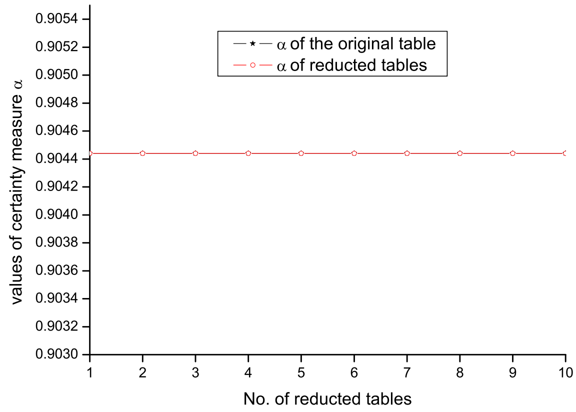

Figure 7. Variation of the certainty measure α after the Liang's entropy reducts.

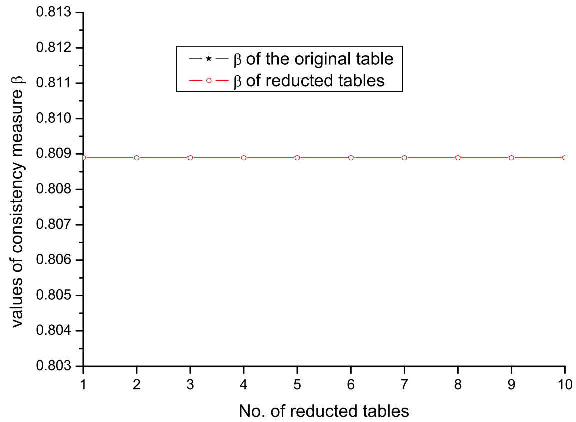

Figure 8. Variation of the consistency measure β after the Liang's entropy reducts.

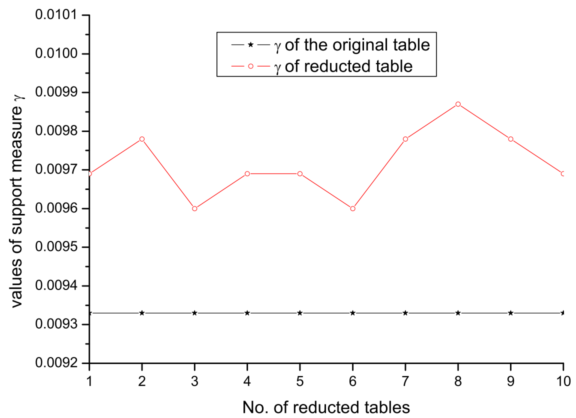

Figure 9. Variation of the support measure γ after the Liang's entropy reducts.

Wei Wei is a PhD candidate from School of Computer and Information Technology at Shanxi University, China. He received the MSc degree in Computers with applications at Shanxi University (2007). His research interests include data mining and knowledge discovery, rough set theory and granular computing.

Jiye Liang is a professor of School of Computer and Information Technology and Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education at Shanxi University. He received the PhD degree in Information Science from Xi'an Jiaotong University. He also has a MS degree in computational mathematics from Xi'an Jiaotong University. His research interests include artificial intelligence, granular computing, data mining and knowledge discovery.

Yuhua Qian is a PhD candidate from School of Computer and Information Technology at Shanxi University, China. He received the MS degree in Computers with applications at Shanxi University. His research interests include data mining and knowledge discovery, rough set theory, granular computing and artificial intelligence.

Feng Wang is a PhD candidate from School of Computer and Information Technology at Shanxi University, China. Her research interests include rough set theory and granular computing.

Chuangyin Dang is a associate professor of the Department of Manufacture Engineering and Engineering Management at City University of Hong Kong. He received his PhD degree (Cum Laude) in Operations Research/Economics from Tilburg University, the Netherlands, and MS degree in Applied Mathematics from Xidian University, China. His research is in operations research/optimization and related fields.